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Indirect taxes and price formation - a model for the Polish economy

1. Introduction

This study aims at extending the elementary input-output price model to represent the role of indirect taxes (formulation of the input-output price model can be found e.g. in R. E. Miller & P. D. Blair [1985]). The proposed extension is mainly based on the solutions presented by R. Bardazzi & M. Grassini [1991] and M. Grassini [1997]. The final formulation meets the problem of unavailability of detailed data on tax components in individual cells of input-output tables, which is likely not only the Polish-specific difficulty.

Building such a model was dictated mainly by some casual needs, i.e. examining effects of possible import tax introduction and effects of adapting VAT rates to the EU standards. However, it can also function as a block of the IMPEC model of the Polish economy.

2. Indirect taxes and price formation - extension of the elementary input-output price model

An important feature of the presented extension is that it is based on the structure of the actually available input-output data, which - in the case of Poland - conform with principles of the System of National Accounts (SNA). Thus, for transparency of further considerations it is necessary at this place to focus on selected data issues and price concepts.

In the SNA, the two main categories of prices are used, i.e. basic prices and final prices (see *The System of National Accounts vol. II* [1997]). Basic prices are defined as amounts of money received by producers for their products, not counting any taxes and subsidies, as well as trade and transport margins. Final prices are amounts as seen from consumers' point of view, i.e. they are inclusive of all taxes and margins. The relation between basic and final prices can be presented as follows (see Zienkowski [2002]):

basic price

\+ customs duties and other import charges

\+ excise and taxes on selected services

(gambling, lottery etc.)

– subsidies

\+ VAT

\+ trade margin

\+ transport margin

= final price (purchaser's price, market price).

According to principles of the SNA, intermediate and final consumption are evaluated in final prices, whereas global output, imports and exports - in basic prices.

Referring to the quoted definitions, global output of a given branch in basic prices is obtained by adding up material costs - evaluated in final prices - and value added of that branch. As a consequence, basic price of a given product is a function of, among other things, all indirect taxes paid on products and services used in the production process.

Among indirect taxes, the role of VAT in price formation requires a closer consideration. "The producer's tax liability is given by the difference between the tax charged on his sales and that paid on his purchases of intermediate goods and services" (see R. Bardazzi and M. Grassini [1991]). In such case, VAT on intermediate products and services makes in fact no cost to the producer, while it can be fully deducted from his tax liability. In terms of prices, VAT does not affect basic price but rather the final price and, therefore, can be treated as a tax on final consumption. However, often there are departures from such a "pure" VAT system, i.e. there are rules limiting deduction of tax paid on producer's inputs. R. Bardazzi and M. Grassini [1991] mention three typical reasons for which deduction of VAT is restrained in most of the EU countries. Firstly, sectors selling products or services which are exempted from VAT (in Poland: education, health care, public administration services, financial services etc.) have no right to deduct the tax. Also, small firms, which do not exceed a certain limit of turnover, can benefit from VAT exemption, regardless of product or service they offer. Secondly, the general rule is that VAT can be deducted only for those products and services which are strictly connected with the output of a given sector. Since such classification of intermediate goods is often difficult, especially for unincorporated family firms, special rules are usually applied, for example the rules limiting deduction of VAT paid on particular goods (e.g. fuels). Finally, for small business there usually exist simplified and standardised methods of tax settlement. In such cases the deductible VAT is established basing on some fixed economic parameters rather than the actual value of intermediate goods and services used in production (e.g. for farmers). In all of those cases, non-deductible VAT takes part in the formation of basic prices and, as well as other indirect taxes, should be present in the extended price equation.

Apart from tax elements, the extended model must also take into account the existence of imports in intermediate use of products and services. Obviously, changes in costs of domestic production should not affect import prices in the model (unless exchange rates are considered), otherwise price effects of the cost-push inflationary spiral could be overestimated in simulations. Therefore, in the proposed model, use of imported products and services is distinguished from that of domestic ones.

The above remarks lead to formulating the input-output price model extended with indirect taxes and margins. Assume that an economy's output can be divided into n groups of homogeneous products and services. Denote by x_{ij} $(i, j = 1, ..., n)$ the value of outlays on intermediate products and services of type *i* used in production of goods of type *j* (x_{ij} are, thus, elements of the first part of a product-to-product input output table). Denote further by X_j $(j = 1, ..., n)$ the global production of goods or services of type *j*. Global output of a given type is equal to material costs augmented by costs of primary production factors (value added), i.e.:

$$X_j = \sum_{i=1}^{n} x_{ij} + V_j \tag{1}$$

where V_j stands for value added associated with goods and services of type *j*.

According to the SNA rules of evaluating transactions, as well as some simplifying assumptions concerning taxes and margins, global output and intermediate use are given as follows:

$$X_j = Q_j \tilde{p}_j^{(d)} \tag{2}$$

and:

$$x_{ij} = \left(q_{ij}^{(d)} \tilde{p}_i^{(d)} \left(1 + s_i^{(d)} + d_i\right) + q_{ij}^{(m)} \tilde{p}_i^{(m)} \left(1 + s_i^{(m)} + d_i\right)\right)\left(1 + h_{ij} t_i\right) \tag{3}$$

where Q_j stands for global output in quantity terms,, $q_{ij}^{(d)}$ and $q_{ij}^{(m)}$ represent quantities of goods of type *i* used as intermediate inputs in production of branch *j*, domestic and imported, respectively, $\tilde{p}_j^{(d)}$ - basic prices of domestic goods, $\tilde{p}_j^{(m)}$ - basic prices of imported goods. Symbol $s_i^{(d)}$ stands for average rate of all indirect taxes except VAT (net - tax minus subsidy rates) paid on domestic goods of type *i* (i.e. excise tax and special taxes on selected services), $s_i^{(m)}$ - average rate of all indirect taxes except VAT (net) paid on imported goods of type *i* (i.e. duties and other import charges as well as excise tax, minus subsidies), $d_i = b_i + c_i$, where b_i is an average rate of trade margin, c_i - average rate of transport margin. Symbol t_i stands for average nominal VAT rate for products or services of type *i* (the same for domestic and imported goods), h_{ij} - a coefficient showing what part of an individual intermediate purchase (in terms of value including all indirect taxes except VAT) is subject to non-deductible VAT. In other words, $h_{ij} t_i$ show rates which can be named as "effective" VAT rates for individual inputs of intermediate products and services.

As can be seen from equation (3), all indirect taxes are assumed to be *ad valorem* taxes with fixed rates. Similarly, trade and transport margins are considered fixed in proportion to value of products and services (in basic prices) belonging to a given group. Such treatment of taxes is perhaps not fully adequate, as it does not represent the actually existent non-linearities in calculation of taxes. This restriction can be, though, justified by the fact that most of the considered indirect taxes are *ad valorem* taxes. Rates are named "average", since individual groups of products or services in many cases are not homogeneous and, thus, within a given group there may coexist products or services taxed at different nominal rates. Actually, the simplifying assumptions - explicit in the formulation of the model - are strictly connected with the structure of the usually available input-output data.

Regarding (2) and (3), cost equation (1) can be written as:

$$Q_j \tilde{p}_j^{(d)} = \sum_{i=1}^{n} \left(q_{ij}^{(d)} \tilde{p}_i^{(d)} \left(1 + s_i^{(d)} + d_i\right) + q_{ij}^{(m)} \tilde{p}_i^{(m)} \left(1 + s_i^{(m)} + d_i\right)\right)\left(1 + h_{ij} t_i\right) + V_j \tag{4}$$

Dividing by Q_j yields:

$$\tilde{p}_j^{(d)} = \sum_{i=1}^{n} \left(\frac{q_{ij}^{(d)}}{Q_j} \tilde{p}_i^{(d)} \left(1 + s_i^{(d)} + d_i\right) + \frac{q_{ij}^{(m)}}{Q_j} \tilde{p}_i^{(m)} \left(1 + s_i^{(m)} + d_i\right)\right)\left(1 + h_{ij} t_i\right) + \frac{V_j}{Q_j} \tag{5}$$

Denoting:

$$\tilde{a}_{ij}^{(d)} = \frac{q_{ij}^{(d)}}{Q_j} \tag{6}$$

$$\tilde{a}_{ij}^{(m)} = \frac{q_{ij}^{(d)}}{Q_j} \tag{7}$$

and:

$$\widetilde{v}_j = \frac{V_j}{Q_j} \tag{8}$$

equation (5) can be then written as:

$$\widetilde{p}_j^{(d)} = \sum_{i=1}^{n} \left(\widetilde{a}_{ij}^{(d)} \widetilde{p}_i^{(d)} \left(1 + s_i^{(d)} + d_i\right)\left(1 + h_{ij} t_i\right) + \widetilde{a}_{ij}^{(m)} \widetilde{p}_i^{(m)} \left(1 + s_i^{(m)} + d_i\right)\left(1 + h_{ij} t_i\right)\right) + \widetilde{v}_j \tag{9}$$

where $\widetilde{a}_{ij}^{(d)}$ and $\widetilde{a}_{ij}^{(m)}$ are technical coefficients showing - in physical terms - the amount of products or services of type i - domestic and imported, respectively - necessary to supply a unit of products or services of type j. Finally, if "$\circ$" stands for element-by -element multiplication, equation (9) can be written in the matrix form:

$$\widetilde{\mathbf{p}}_d = \left(\widetilde{\mathbf{A}}'_d \circ \left(\mathbf{J} + \mathbf{H}'\,\mathbf{T}\right)\right)\left(\mathbf{I} + \mathbf{S}_d + \mathbf{D}\right)\widetilde{\mathbf{p}}_d + \left(\widetilde{\mathbf{A}}'_m \circ \left(\mathbf{J} + \mathbf{H}'\,\mathbf{T}\right)\right)\left(\mathbf{I} + \mathbf{S}_m + \mathbf{D}\right)\widetilde{\mathbf{p}}_m + \widetilde{\mathbf{v}} \tag{10}$$

which after solving yields:

$$\widetilde{\mathbf{p}}_d = \left(\mathbf{I} - \left(\widetilde{\mathbf{A}}'_d \circ \left(\mathbf{J} + \mathbf{H}'\,\mathbf{T}\right)\right)\left(\mathbf{I} + \mathbf{S}_d + \mathbf{D}\right)\right)^{-1} \cdot \left(\left(\widetilde{\mathbf{A}}'_m \circ \left(\mathbf{J} + \mathbf{H}'\,\mathbf{T}\right)\right)\left(\mathbf{I} + \mathbf{S}_m + \mathbf{D}\right)\widetilde{\mathbf{p}}_m + \widetilde{\mathbf{v}}\right) \tag{11}$$

where $\widetilde{\mathbf{p}}_d = \left[\widetilde{p}_i^{(d)}\right]$, $\widetilde{\mathbf{p}}_m = \left[\widetilde{p}_i^{(m)}\right]$, $\widetilde{\mathbf{A}}_d = \left[\widetilde{a}_{ij}^{(d)}\right]$, $\widetilde{\mathbf{A}}_m = \left[\widetilde{a}_{ij}^{(m)}\right]$, $\mathbf{H} = \left[h_{ij}\right]$, $\mathbf{S}_d$ is a diagonal matrix of elements $s_i^{(d)}$, $\mathbf{S}_m$ - diagonal matrix of elements $s_i^{(m)}$, $\mathbf{T}$ - diagonal matrix of elements t_i, $\mathbf{D}$ - diagonal matrix of elements d_i, **I** is a unitary matrix, **J** - matrix with all elements equal 1 $\left(i, j = 1, ..., n\right)$.

It is obvious, however, that input-output tables in physical terms are practically unavailable and, thus, technical coefficients can not be obtained. Instead, value-based coefficients can be used in price equation, which is in fact equivalent to assuming that initially all prices equal 1. Under such assumption, solving price formula leads to obtaining price indices p_i rather than levels $\widetilde{p}_i$ (see R. E. Miller and P. D. Blair [1985]). Define :

$$a_{ij}^{(d)} = \frac{z_{ij}^{(d)}}{X_j} \tag{12}$$

$$a_{ij}^{(m)} = \frac{z_{ij}^{(m)}}{X_j} \tag{13}$$

and:

$$v_j = \frac{V_j}{X_j} \tag{14}$$

where:

$$z_{ij}^{(d)} = q_{ij}^{(d)} \tilde{p}_i^{(d)} \tag{15}$$

and:

$$z_{ij}^{(m)} = q_{ij}^{(m)} \tilde{p}_i^{(m)} \tag{16}$$

where $z_{ij}^{(d)}$ and $z_{ij}^{(m)}$ represent intermediate consumption evaluated in basic prices (which can be considered observable at a certain stage of empirical analysis - for details see the next section), $a_{ij}^{(d)}$ and $a_{ij}^{(m)}$ represent value-based input-output coefficients, v_j - value added per unit of output evaluated in basic prices. The final, applicable version of the extended price model can be, thus, written as:

$$\mathbf{p}_d = \left(\mathbf{I} - \left(\mathbf{A}'_d \circ \left(\mathbf{J} + \mathbf{H}'\mathbf{T}\right)\right)\left(\mathbf{I} + \mathbf{S}_d + \mathbf{D}\right)\right)^{-1}\left(\left(\mathbf{A}_m \circ \left(\mathbf{J} + \mathbf{H}'\mathbf{T}\right)\right)\left(\mathbf{I} + \mathbf{S}_d + \mathbf{D}\right)\mathbf{p}_m + \mathbf{v}\right) \tag{17}$$

where $\mathbf{p}_d = \left[p_i^{(d)}\right]$, $\mathbf{p}_m = \left[p_i^{(m)}\right]$, $\mathbf{A}_d = \left[a_{ij}^{(d)}\right]$, $\mathbf{A}_m = \left[a_{ij}^{(m)}\right]$, $\mathbf{v} = \left[v_i\right]$ $(i, j = 1, ..., n)$. Vectors $\mathbf{p}_d$ and $\mathbf{p}_m$ contain price indices for domestic and imported goods, respectively.

Model represented by equation (17) provides a wide range of possible applications. Apart from analyses of price reaction to changes in unitary value added, it enables examining the effects of changes in tax rates ($\mathbf{S}_d$, $\mathbf{S}_m$ and $\mathbf{T}$), as well as effects of fluctuations of import prices ($\mathbf{p}_m$) and changes of trade and transport margin rates ($\mathbf{B}$, $\mathbf{C}$). However, a problem arises of how to obtain the required parameters of model (17) using the available input-output data. The next section is dedicated to a solution of this problem, which involves procedure of input-output table decomposition.

3. Decomposition of the input-output table

The major problem in practical application of the model given by equation (17) is that the necessary parameter matrices ($\mathbf{A}_d, \mathbf{A}_m, \mathbf{S}_d, \mathbf{S}_m, \mathbf{D}, \mathbf{T}, \mathbf{H}$) are not immediately derived from input-output tables. Generally, calculation of these parameters requires information on tax components in individual transactions of final and intermediate use. Such detailed data are usually unavailable. Therefore, in the Polish case, as perhaps for most European countries, application of the proposed price model relies on decomposition of input-output table, based on incomplete data and simplifying assumptions.

Perhaps, the issue of the greatest concern is to determine h_{ij} coefficients, showing contents of non-deductible VAT in individual intermediate consumption transactions. Solution of this problems is fairly ambiguous and, consequently, any specific assumptions are not embodied in the formulation of model (17). Thus, the model in that form is independent of the actually chosen decomposition method. It must be emphasised that the method of decomposition introduced below is not the only possible one. It should rather be considered as presentation of a certain general approach, which can be a subject of further discussion and development.

Decomposition procedure strictly relies on the structure of the input-output table, which - for the Polish case - is presented in table 1 (all calculations presented in this paper are based upon the product-to-product input-output table for the year 2000 was elaborated by the author basing on supply and use tables, provided by the Central Statistical Office, unpublished). For convenience of the analysis, which is anyway focused on the tax effect on prices, the value added is aggregated into one row, i.e. its components are not distinguished in the presented table.

Table 1. Structure of the input-output table'2000 for Poland.

	[1]	[2]	[3]	Row totals
[1]	$[x_{ij}]_{n\times n}$	$[y_{ik}]_{n\times l}$	$[E_i]_{n\times 1}$	$[G_i]_{n\times 1}$
[2]	$[V_j]_{m\times n}$			
Subtotals ([1], [2])	$[X_j]_{1\times n}$			
[3]	$[M_j]_{1\times n}$			
[4]	$[S_j^{(d)}]_{1\times n}$			
[5]	$[S_j^{(m)}]_{1\times n}$			
[6]	$[T_j]_{1\times n}$			
[7]	$[B_j]_{1\times n}$			
[8]	$[C_j]_{1\times n}$			
Column totals	$[G_j]_{1\times n}$			

where y_{ik} represent final demand, l being the number of the distinguished final demand categories (not counting exports, which is treated separately), G_i - global supply of products or services of type i, E_i - exports of products or services of type i, M_j - imports of products or services of type j, $S_j^{(d)}$ - total amount of indirect taxes, other than VAT, paid on domestic products or services of type j, $S_j^{(m)}$ - total amount of indirect taxes, other than VAT, paid on imported products or services of type j, T_j - total amount of non-deductible VAT paid on both domestic and imported products and services of type j, B_j - total amount of trade margin paid on products and services of type j, C_j - total amount of transport margin paid on products and services of type j.

In the input-output table'2000, "transport, storage and communication" services account were divided into two parts – one concerning transport services connected with trade margins (name it "transport-as-margin"), the other – including transport services treated as intermediate use of other sectors, as well as storage and communication services. As a result, global output of the "transport-as-margin" services equals total of the transport margins paid on all products in the economy. Similarly, "trade and repair" were divided into "trade" and "repair", however in the case of trade, its output equals the total of trade margins across all transactions by definition. Such separation of trade and transport is necessary for the model as well as the decomposition procedure to work properly. It is worth noticing that since margins are recorded in expenditures on other products and services, total supply of trade and “transport-as-margin”, equals zero, as do all values in the corresponding rows of the input-output table.

Extracting VAT from the input-output table

At the first stage, VAT is extracted input-output table. At the same time, VAT rates t_i and coefficients h_{ij} - showing contents of non-deductible VAT in intermediate flows - are determined. The procedure is based on the following relation (see also J. C. Collado & F. Sancho [2002] for method of recovering hidden tax rates in input-output tables in which all transactions contain full amount of deductible):

$$T_i = \sum_{j=1}^{n} \frac{t_i h_{ij}}{1+t_i h_{ij}} x_{ij} + \sum_{k=1}^{l} \frac{t_i}{1+t_i} y_{ik} \qquad (18)$$

In (18) we have *n* equations with $n(n+1)$ variables (t_i and h_{ij}). Thus, to obtain those parameters, *a priori* knowledge and/or simplifying assumptions must be applied. For example, consider the sectors whose products and services are fully exempted from VAT. In the case of Poland these are: fishery, financial services, public administration and defence, education, health care and social security services. As they do not have right to VAT deduction, it is definite that all intermediate inputs in these sectors include full amounts of VAT. Thus, coefficients in the corresponding columns of the **H** matrix are all equal 1.

Equation (18) assumes that all final purchases are inclusive of full VAT amount. However, this need not be true for investment. Simplifying, it can be assumed that enterprises fully exempted from VAT pay the tax on investment goods, other do not (it is deducted). Among institutional sectors of the Polish economy, the non-profit institutions, government and financial enterprises can be treated as the performers of most activities exempted from VAT. Consequently, it was assumed that investment outlays of the mentioned institutional sectors include full VAT amounts, while non-financial enterprises and households (household firms) deduct the whole tax. The only exception is that for private expenditures on buildings, which are also recorded as household investment, but VAT cannot be deducted. Thus, for clarity of the solution presented below, investment in buildings were moved to household consumption.

Following the above remarks, equation (18) should be substituted with:

$$T_i = \sum_{j\in\Omega_1} \frac{t_i h_{ij}}{1+t_i h_{ij}} x_{ij} + \sum_{j\in\Omega_2} \frac{t_i}{1+t_i} x_{ij} + \sum_{k\in\Gamma} \frac{t_i}{1+t_i} y_{ik} \tag{19}$$

where $\Omega_1 = \left\{ j_1^{(1)}, j_2^{(1)}, \ldots, j_\omega^{(1)} \right\}$ is a class of subsequent numbers of groups of commodities which are not exempted from VAT, ω being the total number of such products and services, $\Omega_2 = \left\{ j_1^{(2)}, j_2^{(2)}, \ldots, j_{n-\omega}^{(2)} \right\}$ - a class of subsequent numbers of groups of commodities which are fully exempted from VAT, $\Gamma = \left\{ k_1, k_2, \ldots, k_\gamma \right\}$ - a class of subsequent numbers of final expenditure categories fully inclusive of VAT, γ being the total number of such categories. Still, however it is impossible to solve (19) for t_i and h_{ij} without further simplification.

In order to illustrate the approach better, assume for a while that VAT is present only in final demand (except investment of households and non-financial enterprises) and in intermediate expenditures of branches fully exempted from VAT. In terms of h_{ij} coefficients it means that $h_{ij} = 0$ for $j \in \Omega_1$. Thus, we get:

$$T_i = \sum_{j\in\Omega_2} \frac{t_i}{1+t_i} x_{ij} + \sum_{k\in\Gamma} \frac{t_i}{1+t_i} y_{ik} \tag{20}$$

Solving (20) for t_i yields:

$$t_i = T_i \cdot \left(\sum_{j\in\Omega_2} x_{ij} + \sum_{k\in\Gamma} y_{ik} - T_i \right)^{-1} \tag{21}$$

Result of this experiment for the input-output table'2000 are presented in table 2.

Table 2. VAT rates (in %) and h_i coefficients.

i	Products and services	Results of (21)	Reference rates	Destination rates	h_i
1	Agriculture and forestry	0.8	3.0	0.8	0.000
2	Fishery	0.0	0.0	0.0	0.000
3	Mining	27.2	7.1	7.1	0.226
4	Food	6.6	6.0	6.0	0.216
5	Tobacco	10.8	22.0	10.8	0.000
6	Fabrics	18.8	22.0	18.8	0.000
7	Textile	16.8	22.0	16.8	0.000
8	Leather	11.6	22.0	11.6	0.000
9	Wood	16.0	17.7	16.0	0.000
10	Paper	39.6	22.0	22.0	0.093
11	Publishing and printing	8.5	5.1	5.1	0.793
12	Petrol	36.4	21.1	21.1	0.137
13	Chemicals	8.9	19.4	8.9	0.000
14	Rubber and plastic	58.1	22.0	22.0	0.093
15	Other non-metallic	15.3	12.7	12.7	0.031
16	Metal	4 783.5	21.2	21.2	0.090
17	Metal products	38.3	15.5	15.5	0.117
18	Machines	16.1	19.6	16.1	0.000
19	Office machines and computers	22.4	22.0	22.0	0.021
20	Electric machines	72.4	22.0	22.0	0.170
21	Radio and TV devices	16.9	22.0	16.9	0.000
22	Medical and optical devices	11.4	17.3	11.4	0.000
23	Motor vehicles	19.7	22.0	19.7	0.000
24	Other transport equipment	9.7	7.8	7.8	0.159
25	Furniture and other goods	7.0	22.0	7.0	0.000
26	Recycling	-105.6	22.0	22.0	0.007
27	Electricity, gas, water	3.0	7.0	3.0	0.000
28	Construction	6.8	7.0	6.8	0.000
29	Hotels and restaurants	11.5	22.0	11.5	0.000
30	Financial services	0.1	0.1	0.1	0.000
31	Business and real estate services	11.6	22.0	11.6	0.000
32	Public administration and defence	0.0	0.0	0.0	0.000
33	Education	0.0	0.0	0.0	0.000
34	Health care and social security	0.0	0.0	0.0	0.000
35	Other services	3.1	22.0	3.1	0.000
36	Repair	14.5	22.0	14.5	0.000
37	Transport, storage, communication	10.6	22.0	10.6	0.000
38	Trade	0.0	0.0	0.0	0.000
39	Transport-as-margin	0.0	0.0	0.0	0.000

The first column contains rates calculated according to equation (21). Many of those rates prove to be higher than the highest nominal VAT rate actually used in Poland (22%). The only explanation of such results (apart from inaccuracy of data) is that the denominator in formula (21) is to small for certain types of goods and, consequently, it should be augmented by a part of those intermediate costs, which were assumed VAT-free. Thus, it gives an indirect evidence for what is not explicit in the input-output table, that is for the existence of non-deductible not only in costs of the sectors fully exempted.

The above observation is the basis for the further procedure. The idea amounts to searching for such

h_{ij} values (for $j \in \Omega_1$) which would bring down the resulting t_i rates at least to the level of 22%, (the highest nominal rate) thus allowing to find VAT amounts which are definitely present in intermediate flows. In fact, instead of using the terminal level of 22%, one can calculate “reference rates”, basing on valid tax regulations, which assign nominal VAT rates to different kinds of products and services. This sort of calculation is fairly approximate. If within a group of products there are goods charged at different VAT rates, they should be weighted by shares in global supply of those goods. However, reference rates should perhaps be treated as upper bounds for actual effective rates rather than as actual effective rates themselves. This restriction results from the fact that a given good can be usually supplied by either a VAT- paying producer or by a VAT-exempted one. In such a case, the effective VAT rate becomes lower than the nominal (reference) one.

According to the proposed approach, the so called “destination rates” are determined. Destination rates are set to the reference rates for those products and services, for which rates resulting from equation (21) prove higher than the reference ones (bold font). For the remaining commodities, destination rates are set at the level determined by applying formula (21). Destination rates actually compose the **T** matrix (see model (17)).

Given VAT destination rates, it is possible to calculate h_{ij} coefficients, provided that the number of unknown variables is reduced to *n*. This can be obtained by assuming that those coefficients are uniform across rows of the input-output table. It means that in all branches which are not fully exempted from VAT, the same fraction of value of a particular material is charged with the non-deductible tax. Regarding this assumption, equation (19) can be rewritten as:

$$T_i = \sum_{j \in \Omega_1} \frac{t_i h_i}{1 + t_i h_i} x_{ij} + \sum_{j \in \Omega_2} \frac{t_i}{1 + t_i} x_{ij} + \sum_{k \in \Gamma} \frac{t_i}{1 + t_i} y_{ik} \qquad (22)$$

That simplification enables solving (22) for each h_i separately. The solution yields:

$$h_i = \left(T_i + t_i \left(T_i - \sum_{j \in \Omega_2} x_{ij} - \sum_{k \in \Gamma} y_{ik} \right) \right) \cdot \left(\left(t_i + t_{i2} \right) \left(\sum_{j \in \Omega_1} x_{ij} - T_i \right) + t_{i2} \left(\sum_{k \in \Gamma} y_{ik} + \sum_{j \in \Omega_2} x_{ij} \right) \right)^{-1} \qquad (23)$$

where t_i are VAT destination rates, as shown in table 2. Coefficients h_i are set to zero whenever division by zero arises in (23).

Finally elements of **T** and **H** can be set as:

$$t_{ij} = \begin{cases} t_i & \text{for} \quad i = j \\ 0 & \text{for} \quad i \neq j \end{cases} \qquad (24)$$

$$h_{ij} = \begin{cases} h_i & \text{for} \quad j \in \Omega_1 \\ 1 & \text{for} \quad j \in \Omega_2 \end{cases} \qquad (25)$$

It can be seen from table 2 that the highest contents of non deductible VAT are found in intermediate purchases of food, leather, products of publishing, fuels and fabrics.

The last step in which VAT is involved is purifying input-output table of this tax, in order to enable calculation of the remaining parameters. Denoting by w_{ij} intermediate flows purified of VAT and by f_{ik} - final demand purified of VAT, one can formulate the following purification rules:

$$w_{ij} = \frac{1}{1 + h_{ij} t_i} x_{ij} \qquad (26)$$

$$h_{ij} = \begin{cases} \dfrac{1}{1 + t_i} y_{ik}, & \text{for} \quad k \in \Gamma \\ y_{ik}, & \text{for} \quad k \notin \Gamma \end{cases} \qquad (27)$$

It must be emphasised that the proposed method of determining **T** and **H** parameter matrices is based on rather strong assumptions, which can lead to some undesirable effects in the model. Firstly, unaquaintance with the actual effective rates and using reference rates instead can lead to underestimating contents of non-deductible VAT in intermediate flows. Thus, price responses to VAT rate changes in the model can be interpreted as the minimum of which one can be certain, i.e. actual responses are likely to be greater. The only reason for such limitation in the functionality of the model is the lack of data on non-deductible VAT. Secondly, a researcher must still deal with the notion of effective rather than pure nominal rates which often causes difficulties in adequate formulation of simulation scenarios. Thirdly, the rules in the model are rather compatible with rules limiting deduction of VAT on particular goods, while perhaps in the Polish conditions a greater role should be attributed to VAT exemption for small enterprises. Modelling of the latter case, however, complicates the procedure of determining non-deductible VAT in input-output table. As a result, the proposed method leaves a substantial margin for further consideration and, possibly, improvement.

Purifying the input-output table of indirect taxes other than VAT

Next stages of the decomposition procedure are perhaps more transparent than the previous one. To determine diagonal elements of $\mathbf{S}_d$, $\mathbf{S}_m$, $\mathbf{B}$ and $\mathbf{C}$ matrices, the following formulas are used:

$$s_i^{(d)} = \frac{S_i^{(d)}}{X_i} \tag{28}$$

$$s_i^{(m)} = \frac{S_i^{(m)}}{M_i} \tag{29}$$

$$b_i = \frac{B_i}{X_i + M_i} \tag{30}$$

$$c_i = \frac{C_i}{X_i + M_i} \tag{31}$$

The resulting rates are presented in table 3. Negative values of tax rates mean that subsidies exceed taxes.

Table 3. Rates of indirect taxes excluding VAT (in %), margin rates.

i	Products and services	$s_i^{(d)} \cdot 100$	$s_i^{(m)} \cdot 100$	$b_i \cdot 100$	$c_i \cdot 100$
1	Agriculture and forestry	-0.1	8.1	11.8	1.1
2	Fishery	-0.8	0.6	41.8	10.6
3	Mining	0.0	0.1	5.5	9.8
4	Food	4.8	21.1	27.7	0.2
5	Tobacco	118.4	230.5	92.6	0.0
6	Fabrics	0.0	2.6	18.7	1.1
7	Textile	0.0	9.0	44.7	1.6
8	Leather	0.0	7.0	48.6	1.7
9	Wood	0.0	2.4	12.5	1.4
10	Paper	0.0	0.9	13.1	2.1
11	Publishing and printing	-0.1	0.9	19.1	0.8
12	Petrol	30.0	120.5	56.0	3.5
13	Chemicals	-0.1	1.6	26.8	0.6
14	Rubber and plastic	0.1	1.9	15.6	3.9
15	Other non-metallic	0.0	1.5	14.0	7.5
16	Metal	0.0	2.4	5.9	0.5
17	Metal products	0.0	2.7	10.2	3.2
18	Machines	0.0	1.4	8.1	0.5
19	Office machines and computers	0.0	1.1	18.4	2.3
20	Electric machines	0.0	1.4	8.0	0.9
21	Radio and TV devices	0.0	2.2	16.6	1.0
22	Medical and optical devices	0.0	2.5	6.9	2.3
23	Motor vehicles	1.4	8.8	12.3	0.6
24	Other transport equipment	0.0	1.7	7.3	0.5
25	Furniture and other goods	0.0	4.8	13.7	1.8
26	Recycling	0.0	0.0	0.0	1.5
27	Electricity, gas, water	0.0	2.3	0.5	0.0
28	Construction	0.0	0.0	0.0	0.0
29	Hotels and restaurants	-0.1	0.0	0.0	0.0
30	Financial services	0.0	0.0	0.0	0.0
31	Business and real estate services	-0.5	-0.3	0.0	0.0
32	Public administration and defence	0.0	0.0	0.0	0.0
33	Education	0.0	0.0	0.0	0.0
34	Health care and social security	0.0	0.0	0.0	0.0
35	Other services	-1.1	-0.9	0.0	0.0
36	Repair	0.0	0.0	0.0	0.0
37	Transport, storage, communication	-0.9	-0.9	0.0	0.0
38	Trade	0.0	0.0	-100.0	0.0
39	Transport-as-margin	0.0	0.0	0.0	-100.0

Distinguishing between domestic and imported production relies on the assumption, according to which, for particular commodity there is a fixed share of imports, identical in all individual purchases of that commodity, i.e.:

$$z_{ij}^{(m)} = \mu_i z_{ij} \tag{32}$$

where z_{ij} represent intermediate flows evaluated at basic prices (see (15) and (16)):

$$z_{ij} = z_{ij}^{(d)} + z_{ij}^{(m)} \tag{33}$$

Coefficients μ_i can be calculated according to the following rule:

$$\mu_i = \frac{M_i}{X_i + M_i} \tag{34}$$

The above assumption makes it possible to purify transactions of both indirect taxes on domestic goods and those on imported ones, as well as trade and transport margin, at once. The appropriate rule is given by:

$$z_{ij} = \frac{w_{ij}}{1 + b_i + c_i + \mu_i s_i^{(m)} + (1 - \mu_i) s_i^{(d)}} \tag{35}$$

Having z_{ij} and μ_i, it is possible to determine elements of the last two parameter matrices, $\mathbf{A}_d$ and $\mathbf{A}_m$:

$$a_{ij}^{(d)} = (1 - \mu_i) \frac{z_{ij}}{X_j} \tag{36}$$

$$a_{ij}^{(m)} = \mu_i \frac{z_{ij}}{X_j} \tag{37}$$

Finally, all of the acquired parameter matrices should be diminished by removing rows and columns corresponding with trade and "transport-as-margin". Otherwise the model could not be solved as all elements of $\mathbf{A}_d$ and $\mathbf{A}_m$ matrices in rows corresponding with trade and "transport-as-margin" equal zero (margins are present explicitly in the price equation, so the solution remains correct). The test for correctness of the decomposition procedure is done by calculating prices according to formula (17), using the acquired parameter matrices – all of the resulting price indices should equal one.

4. Final prices and aggregate price indices

At the last stage of the analysis, final prices, as well as aggregate price indices are determined.

Final price for a particular group *i* of products or services can be calculated as:

$$r_i^{(d)} = (1+t_i)(1+s_i^{(d)}+d_i)p_i^{(d)} \tag{38}$$

Values of $r_i^{(d)}$ show the relation of the final to the basic price. Changes of final prices of domestic goods are represented by the following indices:

$$\pi_i^{(d)} = r_i^{(d)} / \grave{r}_i^{(d)} \tag{39}$$

where $\grave{r}_i^{(d)}$ stands for initial values of $r_i^{(d)}$, that is the values calculated basing on original parameters of the model (not changed due to scenario assumptions).

Analogously, final prices as well final price indices for particular imported commodities can be calculated, i.e.:

$$r_i^{(m)} = (1+t_i)(1+s_i^{(m)}+d_i)p_i^{(m)} \tag{40}$$

and:

$$\pi_i^{(m)} = \frac{r_i^{(m)}}{\grave{r}_i^{(m)}} \tag{41}$$

For interpretation purposes, it is usually convenient to use a weighted price index for subsequent groups of products and services, including both domestic and imported ones:

$$\pi_i = \frac{(1-\mu_i)r_i^{(d)} + \mu_i r_i^{(m)}}{(1-\mu_i)\grave{r}_i^{(d)} + \mu_i \grave{r}_i^{(m)}} \tag{42}$$

Aggregate price index for category *k* of final expenditures (not exempted from VAT - e.g. final consumption) can be calculated as follows:

$$\lambda_k = \left(\sum_{i=1}^{n} \pi_i y_{ik} \right) \cdot \left(\sum_{i=1}^{n} y_{ik} \right)^{-1} \tag{43}$$

where $k \in \Gamma$.

Values of y_{ik} are taken directly form the input-output table (see table 1).

Aggregate price index of global domestic output is given by:

$$\lambda^{(X)} = \left(\sum_{i=1}^{n} p_i^{(d)} X_i \right) \cdot \left(\sum_{i=1}^{n} X_i \right)^{-1} \tag{44}$$

Define also:

$$\delta_i^{(d)} = \frac{\left(1 + s_i^{(d)} + d_i\right) p_i^{(d)}}{1 + \grave{s}_i^{(d)} + \grave{d}_i} \tag{45}$$

$$\delta_i^{(m)} = \frac{\left(1 + s_i^{(m)} + d_i\right) p_i^{(m)}}{1 + \grave{s}_i^{(m)} + \grave{d}_i} \tag{46}$$

and

$$\delta_i = \frac{(1 - \mu_i)\left(1 + s_i^{(d)} + d_i\right) p_i^{(d)} + \mu_i \left(1 + s_i^{(m)} + d_i\right) p_i^{(m)}}{(1 - \mu_i)\left(1 + \grave{s}_i^{(d)} + \grave{d}_i\right) + \mu_i \left(1 + \grave{s}_i^{(m)} + \grave{d}_i\right)} \tag{47}$$

where $\grave{s}_i^{(d)}$, $\grave{s}_i^{(m)}$ and $\grave{d}_i$ are initial values of indirect tax rates (except VAT) and margins - as determined in the decomposition procedure. Values of δ_i can be treated as final price indices for commodities for which full deduction of VAT is allowed. Thus, it leads to formulating aggregate price indices for those final demand categories which are free of VAT burden (e.g. exports and investment of non-financial enterprises), as well as to the mixed aggregated price index for a wider range of final goods, respectively (e.g. GDP):

$$\lambda_k^{(F)} = \left(\sum_{i=1}^{n} \delta_i y_{ik} \right) \cdot \left(\sum_{i=1}^{n} y_{ik} \right)^{-1} \tag{48}$$

where $k \in \{1,2,...,l\}$ and:

$$\lambda_k^{(A)} = \left(\sum_{k \in U_1} \sum_{i=1}^{n} \pi_i y_{ik} + \sum_{k \in U_2} \sum_{i=1}^{n} \delta_i y_{ik} \right) \cdot \left(\sum_{k \in U_1} \sum_{i=1}^{n} y_{ik} + \sum_{k \in U_2} \sum_{i=1}^{n} y_{ik} \right)^{-1} \tag{49}$$

where $U_1 \subset \Gamma, \quad U_2 \subset \{1,2,...,l\}$.

According to analytical needs, other price indices can be defined.

5. Simulation results

Simulation assumptions were inspired mainly by the two facts. Firstly, it was the discussion among the Polish politics and economists on the possibility of introducing a 5% import tax. Secondly, it is the EU accession, which entails adaptation of VAT rates on certain products and services in the next years. Therefore, the empirical part of the study is divided into two parts, adequate for the analysed issues.

Simulation 1 - import tax

Applying scenario for the first simulation is fairly simple. It amounts to adding 0.05 to each rate of indirect taxes on imported goods $s_i^{(m)}$. The results are presented in tables 4 and 5.

The relatively highest growths of final prices can be observed for office machines, motor vehicles and radio and TV devices, mainly as a result of high import shares for these products. However, final prices are also driven by significant increase in material costs, which is visible in basic prices (for industry rates of growth of basic prices oscillate around 1% - 2%). It is the office-machine industry who bears the highest indirect costs of introducing import tax. Among commodities showing the weakest reaction, as far as prices are concerned, there are essentially those which absorb little material costs. One should mention here mainly education, public administration and defence and health care services.

Table 4. Results of simulation 1. Rates of growth of prices, share of imports in supply (in %).

i	Products and services	Basic prices (domestic)	Final prices (domestic + imported)	Share of imports in total supply
1	Agriculture and forestry	1.0	1.3	10.0
2	Fishery	1.2	1.7	22.9
3	Mining	0.7	2.1	38.9
4	Food	0.9	1.1	8.0
5	Tobacco	0.6	0.7	2.5
6	Fabrics	1.5	2.6	42.4
7	Textile	0.8	1.2	16.7
8	Leather	1.1	1.6	26.1
9	Wood	1.0	1.3	9.9
10	Paper	1.7	2.8	40.6
11	Publishing and printing	0.9	1.1	7.9
12	Petrol	1.6	1.6	15.4
13	Chemicals	1.0	2.5	50.7
14	Rubber and plastic	1.5	2.4	34.6
15	Other non-metallic	1.0	1.6	20.9
16	Metal	1.3	2.2	27.0
17	Metal products	1.2	2.0	25.1
18	Machines	1.3	2.9	50.4
19	Office machines and computers	2.2	3.9	90.5
20	Electric machines	1.6	2.6	35.3
21	Radio and TV devices	1.5	3.1	60.6
22	Medical and optical devices	1.0	2.2	34.7
23	Motor vehicles	1.8	3.1	55.9
24	Other transport equipment	1.6	1.9	9.7
25	Furniture and other goods	1.2	1.6	13.4
26	Recycling	1.5	1.5	0.0
27	Electricity, gas, water	1.0	1.0	0.3
28	Construction	0.9	1.0	2.5
29	Hotels and restaurants	0.6	0.6	0.0
30	Financial services	0.9	1.8	21.2
31	Business and real estate services	0.5	0.7	4.4
32	Public administration and defence	0.1	0.1	0.0
33	Education	0.2	0.2	0.0
34	Health care and social security	0.2	0.2	0.0
35	Other services	0.5	0.5	1.2
36	Repair	0.6	0.6	0.8
37	Transport, storage, communication	0.8	1.5	16.7

Table 5. Results of simulation 1. Rates of growth of aggregate prices (in %).

Categories of final expenditures + global output	**Price change**
Private consumption	1.3
Government consumption	0.3
Investment	2.0
Export	1.9
Global domestic output (basic prices)	0.8

Table 5 shows changes of prices for major categories of final demand, as well as global output. Differences among the rates of growth can be explained, firstly, by differences in expenditure structures across categories. The second reason is that to evaluate global output, basic prices are used, while for the demand – final prices. The highest price increase is that of investment, the lowest - that of government consumption. In general, introducing import tax would rather have unfavourable impact on investment conditions, since relation of prices of investment goods to prices of consumption would worsen. The increase of prices is also comparatively high for exports. On the other hand, however, import tax seems an attractive source of budget revenues. Government consumption becomes relatively cheap, meaning that besides direct outcome in a form of tax revenue increase, additional redistribution effect appears.

Simulation 2 - VAT adaptation to the EU standards

In the second experiment, potential effects of the forthcoming VAT rate changes are analysed. The point of changes is limiting the range of reduced and zero rates (e.g. in construction and agriculture). The major changes in VAT rates are pointed out in table 6.

Table 6. Major forthcoming changes in VAT rates in Poland (rates in %).

Products and services	Current rate	New rate
Agriculture	3	7
Means of agriculture production	3	7
Construction	7	22
Materials for construction	7	22

To set proper values of t_i for simulation, new reference rates were calculated (see the previous section) taking into account the planned changes. Calculating differences between these rates and the old reference rates (see table 2), leads to finding the necessary adjustments of t_i in the simulation. The adjustments are presented in table 7. Tables 8 & 9 show results of simulation 2.

Table 7. Changes of VAT rates in simulation 2 (in percentage points).

i	Products and services	Changes of VAT rates
1	Agriculture and forestry	7.0
9	Wood	4.3
11	Publishing and printing	2.3
13	Chemicals	0.1
15	Other non-metallic	9.3
16	Metal	0.8
17	Metal products	6.5
18	Machines	0.5
25	Furniture and other goods	0.3
27	Construction	15.0

In the case of VAT increase, price changes are predominated by direct effects, visible in final prices. However, a slight reaction of basic prices can also be observed - generally being at the level of 0.1% - 0.2%. For products and services fully exempted from VAT this reaction is reasonably stronger. For example, growths of basic prices of fishery products exceeds 1%, while for public administration and defence, education, as well as publishing and printing, these rates are close to 0.5%. As far as aggregate categories are concerned, it is consumption that goes up the highest, for other categories, price reaction is very slight. Again, as in the case of import tax, the state budget additionally benefits from relative price changes, but the cost this time is not transferred neither to investment nor exports.

It can be said that in the case of increase of the considered VAT rates (agriculture products, construction services, materials for agriculture, materials for construction, publishing and printing products), the role of indirect cost-push effects in price formation proved rather insignificant. It means that VAT on these particular products have little share in the non-deductible VAT in the economy. can be mostly deducted by producers, being rather lucky coincidence as far as inflation in Poland is concerned. One should have in mind, however, that the currently proposed procedure of purifying input-output table of VAT is imperfect and may lead to underestimating the role of non-deductible VAT in formation of basic prices. Thus, this is the main indication the main point in further development of the price model.

Table 8. Results of simulation 2. Rates of growth of prices (in %).

i	Products and services	Basic prices (domestic)	Final prices (domestic + imported)
1	Agriculture and forestry	0.0	7.0
2	Fishery	1.1	0.9
3	Mining	0.1	0.0
4	Food	0.1	0.1
5	Tobacco	0.1	0.1
6	Fabrics	0.0	0.0
7	Textile	0.0	0.0
8	Leather	0.1	0.0
9	Wood	0.1	3.8
10	Paper	0.0	0.0
11	Publishing and printing	0.5	2.7
12	Petrol	0.0	0.0
13	Chemicals	0.0	0.1
14	Rubber and plastic	0.0	0.0
15	Other non-metallic	0.1	8.3
16	Metal	0.1	0.7
17	Metal products	0.2	5.7
18	Machines	0.1	0.5
19	Office machines and computers	0.0	0.0
20	Electric machines	0.1	0.1
21	Radio and TV devices	0.0	0.0
22	Medical and optical devices	0.1	0.1
23	Motor vehicles	0.2	0.1
24	Other transport equipment	0.1	0.1
25	Furniture and other goods	0.1	0.3
26	Recycling	0.1	0.1
27	Electricity, gas, water	0.0	0.0
28	Construction	0.1	14.2
29	Hotels and restaurants	0.0	0.0
30	Financial services	0.2	0.1
31	Business and real estate services	0.1	0.1
32	Public administration and defence	0.4	0.4
33	Education	0.4	0.4
34	Health care and social security	0.3	0.3
35	Other services	0.1	0.1
36	Repair	0.0	0.0
37	Transport, storage, communication	0.0	0.0

Table 9. Results of simulation 2. Rates of growth of aggregate prices (in %).

Categories of final expenditures + global output	**Price change**
Private consumption	1.7
Government consumption	0.3
Investment	0.1
Export	0.1
Global domestic output	0.1

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