The Craft of Economic Modeling

Part III. Multisectoral Models

Input-Output Flow Table

Seller \ Buyer	Agriculture	Mining	Gas & Elec	Manufac-turing	Com-merce	Trans-port	Ser-vices	Govt. Ind	Consump-tion	Govern-ment	Invest-ment	Exports	Imports	Final Demand	Row Sum
Agriculture	20	1	0	100	5	0	2	0	15	1	0	40	-20	36	164
Mining	4	3	20	15	2	1	2	0	2	1	0	10	-10	3	50
Gas&Electric	6	4	10	40	20	10	25	0	80	10	0	0	0	90	205
Mfg	20	10	4	60	25	18	20	0	400	80	200	120	-170	630	787
Commerce	2	1	1	10	2	3	6	0	350	10	6	10	0	376	401
Transport	2	1	5	17	3	2	5	0	130	20	8	5	0	163	198
Services	6	3	8	45	20	5	20	0	500	40	10	30	-20	560	667
GovInd	0	0	0	0	0	0	0	0	0	150	0	0	0	150	150
Intermediate	60	23	48	287	77	39	80	0							614
Deprec.	8	4	40	40	25	30	20	0							167
Labor	68	21	31	350	150	107	490	150							1367
Capital	20	2	56	60	40	12	59	0							259
Indirect tax	8	0	20	50	109	10	18	0							215
Value added	104	27	147	500	324	159	587	150							2008
ColSum	164	50	205	787	401	198	667	150	1477	312	224	215	-220	2008	

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Part III. Multisectoral Models

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CHAPTER 14. INPUT-OUTPUT IN THE IDEAL CASE

14.1. Input-Output Flow Tables

Multisectoral models begin from an accounting of the flows of goods and services among various industries of the economy. Table 14.1 shows a simple interindustry accounting, or input-output flow table, for an imaginary but not unrealistic eight-sector economy which we will call *Tiny*. The simplicity is to make it easy for us to concentrate on essential concepts without being overwhelmed by big tables of data. In Table 14.1, the selling industries are listed down the left side of the table. The last industry, abbreviated as "GovInd," is "Government Industry", a fictitious industry which simply supplies the government with the services of its own employees. Below these come the classes of factor payments: Depreciation, Labor compensation, Capital income (such as interest, profits, rents, or proprietor income), and Indirect taxes (such as property taxes, sales taxes, and excise taxes as on alcohol, tobacco, and gasoline). Note the similarity of these categories of factor payments to the categories of national income. Their sum is the row named Value added. Across the top of the table the same eight industries are listed as buyers of products. Here they are followed by columns corresponding to the principal divisions of the "product side" of the national accounts:

- Con - Personal consumption expenditure
- Gov - Government purchases of goods and services
- Inv - Investment
- Exp - Exports
- Imp - Imports (as negative numbers)

In input-output terms, these are the final demand columns. The next-to-last column, labeled FD for "Final Demand," shows their sum. It is shaded to emphasized that it is derived by summing other columns. The next last column, also shaded, is the sum of all the (non-shaded) elements row.

Across each row of the table the sales of that industry to each of the industries and final demand columns are shown. Thus, the 100 in the Agriculture row and Manufacturing (Mfg) column means that Agriculture sold 100 billion dollars (bd) of products to Manufacturing in the year covered by this table. Typical sales here are grains to milling, live animals to meat packing, or fruits and vegetables to plants which can or freeze them. The 15 in the Personal consumption (Con) column of the same row means that Agriculture sold 15 bd of products directly to households during the year. These sales are primarily fresh fruits and vegetables and eggs. In the table shown here, which is in producer prices, agricultural products are recorded at the price the farmer received for them. These products are not necessarily bought at the farm gate. Going through wholesale and retail trade channels does not change the industry of origin of a product; going through a manufacturing process does. Thus, an orange sold as an orange to she who eats it appears as a sale from Agriculture to Personal consumption, despite the fact that it was purchased in a store. Another orange that was turned into frozen orange juice appears first as a sale from Agriculture to Manufacturing at the price received by the farmer. It then reappears as a sale from Manufacturing to Personal consumption at the manufacturer's price. Yet the price paid by the ultimate consumer is neither the price received by farmer in the first case nor by the manufacturer in the second. Where is the difference, the commercial margin? In this table, it is in the sales of Commerce to Personal consumption expenditure. Transportation margins are handled similarly. Tables made with this pricing

convention are said to be "in producer prices". We shall look at other ways of handling the problem of margins in Chapter 16.

Table 14.1. Input-Output Flow Table

Seller \ Buyer	Agriculture	Mining	Gas & Elec	Mfg	Com-merce	Trans-port	Ser-vices	Gov Ind	Con	Gov	Inv	Exp	Imp	FD	Row Sum
Agriculture	20	1	0	100	5	0	2	0	15	1	0	40	-20	36	164
Mining	4	3	20	15	2	1	2	0	2	1	0	10	-10	3	50
Gas&Electric	6	4	10	40	20	10	25	0	80	10	0	0	0	90	205
Mfg	20	10	4	60	25	18	20	0	400	80	200	120	-170	630	787
Commerce	2	1	1	10	2	3	6	0	350	10	6	10	0	376	401
Transport	2	1	5	17	3	2	5	0	130	20	8	5	0	163	198
Services	6	3	8	45	20	5	20	0	500	40	10	30	-20	560	667
GovInd	0	0	0	0	0	0	0	0	0	150	0	0	0	150	150
Intermediate	60	23	48	287	77	39	80	0							614
Deprec.	8	4	40	40	25	30	20	0							167
Labor	68	21	31	350	150	107	490	150							1367
Capital	20	2	56	60	40	12	59	0							259
Indirect tax	8	0	20	50	109	10	18	0							215
Value added	104	27	147	500	324	159	587	150							2008
ColSum	164	50	205	787	401	198	667	150	1477	312	224	215	-220	2008	

As we look down the column for an industry, we see all the products which it needs for making its own. In the Agriculture column, we see first of all 20 bd from Agriculture itself. These are sales primarily of feed grains to animal husbandry, but include also sales of seed, hay, manure, and other products. These sales within the industry are common and are referred to in input-output jargon as "diagonals" because they appear on the main diagonal of the table. Further down the Agriculture column we see 4 bd for Mining, primarily crushed limestone, but also some coal. The 20 bd spent on Manufacturing bought gasoline, fertilizers, and pesticides. The 2 bd spent on Commerce were trade margins on these manufactured products. The 2 bd spent on Transport included transportation margins on the products of the other industries as well as costs incurred by the farmer in getting products to market. The purchases from Services includes the services of veterinarians, lawyers, and accountants. All the purchases of the industries from each other are called "intermediate" purchases because they do not go directly to the final user but are "mediated" by other industries. The sum of the intermediate purchases by each industry are in the row labeled "Intermediate" and shaded, as before, to show that it is derived by adding other entries in the table.

Below the "Intermediate row" are the value-added rows. We find that Depreciation of equipment came to 8 bd. Labor received 68 bd. (In our imaginary economy, we imagine that proprietor income has been divided between labor and capital income. In most actual tables, it will be shown separately or classified as capital income.) The 20 bd of capital income includes interest payments, corporate profits, and capital's portion of proprietor income. The 8 bd of Indirect taxes is mostly property taxes.

The Capital income row of value added includes both corporate profits and proprietor income. Since it is the total of sales minus the total of expenses, the column sum for each industry is equal to its row sum. For example, the row sum of Agriculture is 164 and the column sum (of the unshaded

entries) is 164, and so on for all eight industries. This fact has a remarkable consequence which is the cornerstone of national accounting, namely that the sum of all the value-added entries is equal to the sum of all the final demand entries. In our table, each of these groups of entries is surrounded by a double line and each adds to 2008. Why is the total the same? Since the sum of each of the eight industry rows, say R, is equal to the sum of the corresponding column, the sum of all eight rows, 2622, is equal to the sum of all eight columns, say C, which is also 2622. Thus we have with R = C. But the total of the final demands, D, is R minus the total of the intermediate flows, say X, or D = R - X. Likewise, the total value added, V, is C, the sum of all the industry columns, less the sum of that part of them which is intermediate, or V = C - X. But R = C implies that R - X = C - X or D = V. Naturally, this D or V has a name, and that name is Gross Domestic Product. We have thus proved the fundamental identity of national accounting: Gross Domestic Product (GDP) is the same whether measured by the products that go to final demand or by the income which goes to factors. In our table, this identity appears in the fact that the sum of the FD column, 2008, is the sum of the Value added row, also 2008, which is the GDP of this economy. Arrayed in format of national accounts, our economy would appear as in Table 14.2.

Table 14.2 The Income and Product Account

Gross domestic product	2008	Gross domestic product	2008
Personal consumption	1477	- Depreciation	167
Investment	224	= Net domestic product	1841
Exports	215	- Indirect taxes	215
Imports	-220	= National income	1626
Government purchases	312	Labor income	1367
		Capital income	259

Before leaving Table 14.1, we must make a fundamental point about it. With one small exception, the table makes sense in physical units. We can measure the output of Agriculture in bushels, that of Mining in tons, that of Gas and Electricity in BTU's, Transport in ton-miles, Labor in worker hours, Capital income in ounces of gold, and so on. Wassily Leontief, maker of the first input-output table, used to often insist in seminars that any calculations had to make sense in physical terms[1].

The small exception, however, is important: the column sums of a table in physical terms are utterly meaningless since all the elements are in different units. Naturally, the row totals -- which are meaningful -- do not equal the meaningless totals of the corresponding columns. This point would seem so obvious as to be not worth making were it not for the fact that it is often forgotten, precisely by the makers of input-output tables. For if a table is made in the prices of some year other than the year to which it refers, it is essentially in physical units. Thus, we can make a table for 2000 in 1980 prices, where the physical measure in each row is "one 1980 dollar's worth" of the product. In other words, the physical unit for each product is how much of it one dollar would buy in 1980. For any product for which a price index can be made, 2000 dollar amounts can be converted into 1980 dollar physical units by the price index. For value added, since there is no very natural unit, one can simply deflate all of the value-added cells by the GDP deflator. The total real value added will then be the same as total real final demand. One can have in this way a perfectly sensible, meaningful

1 In fact, tables in physical terms have been developed for several countries, and are essential to the study of materials flows.

table. *But its column sums are meaningless and certainly do not equal the corresponding row sums.*

Unfortunately, some table makers have disregarded this fact and have simply forced the value added in each industry of such a table to equal the difference between the row sum of the industry and the sum of the intermediate inputs into it. The results make as much sense as saying that five squirrels minus three elephants equals two lions. The arithmetic is right but the units are crazy.

This practice is called "double deflation" because first the outputs are deflated and then the purchased inputs deflated and subtracted from the deflated output to obtain a mongrel, mixed-up-units number, possibly positive but also possibly negative, mistakenly alleged to be a measure of "constant-price value added". It is indeed what would have been left over for paying primary factors, had producers gone right on producing with the previous period's inputs after prices have changed. That is certainly no measure of "real value added," for it is not, in all probability, what producers did. The error would perhaps be easier to see if labor input, for which we have some measures of cost, were considered as an intermediate input and indirect taxes were simply subtracted in current prices from output. The double-deflation procedure should then give a measure of "real capital income." In such a table, the deflators for capital income would be different in different industries. The residuals might well be negative, especially if there were a few years between the two periods. Trying to deflate the difference between two numbers that are very close together by deflating each of the two numbers by different deflators and then taking the difference between the two deflated items is simply asking for trouble.

The difficulties due to double deflation are often masked by the taking the time periods of the tables close together and "chaining" the index, so that negative values are unlikely. But the calculation still really does not make sense. Unfortunately, these procedures are sanctioned by international statistical standards, and many statistical offices engage in them. Economists have made matters worse by taking these mixed-units numbers as measures of "real" product in studies of productivity.

As far as I am aware, there is no satisfactory way of measuring real productivity at the individual industry level, precisely because industries cooperate with one another in production, and how they do so changes. In one year, for example, the "television set industry" is a collection of plants that make the cabinets, the tubes and the electronics, and assemble the sets. In a later year, the industry has become assembly plants that buy cabinets, tubes, and electronics and assemble them. Clearly, changes in sales (even in constant prices) divided by labor input in worker hours in this one industry is not an appropriate measure of productivity increase. Rather, changes in "productivity" in this case is meaningful only as applied to how much labor and capital is required by the whole economy to produce a television set. We shall see how it can be meaningfully calculated. The meaningful, correct calculation has nothing whatever to do with double deflation. But the quest to allocate the changes in whole-economy productivity for particular products to individual industries is a search for a nonexistent – and superfluous – El Dorado[2].

2 A mythical "city of gold", searched for by Sir Walter Raleigh and many Spanish explorers.

14.2. Input-Output Equations. The Fundamental Theorem.

An input-flow table describes an economy in a particular year. Its greatest value, however, lies in the ability it gives us to answer the question What would the outputs, value added, and intermediate flows have been had the final demands been different? To answer that question in the simplest possible way, we must assume that the ratio of each input into an industry to that industry's output remains constant when the final demands are changed. These ratios are known as the "input-output coefficients," and may be defined by

$$a_{ij} = \frac{x_{ij}}{q_j}$$

where x_{ij} is the flow from industry i to industry j in Table 14.1 and q_j is the output of industry j, that is, it is the sum of row j or column j in the same table. For example,

$$a_{1,4} = \frac{100}{787} = 0.12706$$

Table 14.3 shows the complete matrix of these input-output coefficients corresponding to Table 14.1.

Table 14.3. Input-Output Coefficients

	Agric	Mining	Gas&Elec	Mfg	Com	Trans	Serv	GovInd
Agriculture	0.1220	0.0200	0.0000	0.1271	0.0125	0.0000	0.0030	0.0000
Mining	0.0244	0.0600	0.0976	0.0191	0.0050	0.0051	0.0030	0.0000
Electricity	0.0366	0.0800	0.0488	0.0508	0.0499	0.0505	0.0375	0.0000
Manufacturing	0.1220	0.2000	0.0195	0.0762	0.0623	0.0909	0.0300	0.0000
Commerce	0.0122	0.0200	0.0049	0.0127	0.0050	0.0152	0.0090	0.0000
Transportation	0.0122	0.0200	0.0244	0.0216	0.0075	0.0101	0.0075	0.0000
Services	0.0366	0.0600	0.0390	0.0572	0.0499	0.0253	0.0300	0.0000
GovInd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

If we are willing to suppose that these coefficients remain constant as the final demand vector changes, then for any vector of final demands, *f*, we can calculate the vector of industry outputs, *q* from the equation

$$q = Aq + f \qquad (14.2.1)$$

where *A* is the square matrix of input-output coefficients in Table 14.3. If we happen to choose as *f* the column vector of final demands in Table 14.1, (the first eight elements of the FD column: (36,3,90, ..., 150)'), then *q* should be the column vector of industry outputs of Table 14.1 (the vector of row sums of the eight industry rows: (164,50,205,...,150)'). For other values of *f*, of course, we will find other values of *q*.

One way of solving (14.2.1) is to rewrite it as

$$(I - A)q = f$$

or

$$q = (I - A)^{-1} f$$

The matrix of $(I - A)^{-1}$ on the right of this equation is known as the *Leontief inverse* of the A

matrix. For our example, it is shown in Table 14.4. Its elements have a simple meaning. Element (i,j) shows how much of product i must be produced in order to produce one unit of final demand for product *j*. This interpretation is readily justified by taking *f* to be a vector of zeroes except for a 1 in row *i*. Then *q* will be the *i*th column of $(I - A)^{-1}$, and its *j*th element will show exactly how much of product *j* will have to be produced in order to supply exactly one unit of *i* to final demand. In our example, in order to supply one unit of Agricultural product to final demand, 0.1691 units of Manufacturing must be produced. Note that, in the example, all elements of the Leontief inverse are non-negative. In view of the economic interpretation, that result is hardly surprising. Later in this chapter, we will show mathematically that the Leontief inverse from an observed A matrix is always non-negative.

Table 14.4. The Leontief Inverse $(I-A)^{-1}$

	Agri.	Mining	Gas&El.	Mfg.	Comm.	Transport	Services	Govt Ind.
Agriculture	1.1647	0.0620	0.0107	0.1634	0.0263	0.0165	0.0096	0.0000
Mining	0.0405	1.0830	0.1126	0.0352	0.0144	0.0150	0.0092	0.0000
Gas & Electric	0.0617	0.1137	1.0683	0.0748	0.0623	0.0641	0.0452	0.0000
Manufacturing	0.1691	0.2530	0.0538	1.1201	0.0791	0.1091	0.0396	0.0000
Commerce	0.0184	0.0276	0.0093	0.0185	1.0077	0.0180	0.0106	0.0000
Transport	0.0210	0.0319	0.0304	0.0297	0.0120	1.0151	0.0102	0.0000
Services	0.0604	0.0911	0.0548	0.0791	0.0612	0.0379	1.0368	0.0000
Govt Industry	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

We may also ask how much of a primary resource, such as Labor or Capital, would be needed for the production of a given final demand. We may define the resource coefficients similarly to the input-output coefficients by

$$r_{ij} = \frac{y_{ij}}{q_j}$$

where y_{ij} is the payment to factor *i* by industry *j*. For example, from Table 14.1, $y_{2,4}$, the payment to resource 2, Labor, by industry 4, Manufacturing, is 360. If we denote by *R* the matrix of the r_{ij}, then the vector of total payments to each resource for an output vector *q* is *Rq*, and for a final demand vector, *f*, it is

$$R(I-A)^{-1} f \quad .$$

If we now think of each row of this matrix as a row vector and sum these vectors – a process which makes sense if all the rows are measured in monetary values in the prices of the year of the table – we get a row vector *v* of value-added per unit of output. Just as previously we asked how output *q*, would change if *f* changed while *A* remains constant, we can now ask how prices *p* would change if *v* changed while *A* remains constant. The row vector *p* must satisfy the equations

$$p = pA + v \tag{14.2.2}$$

These equations state simply that the price of a unit of each product is equal to the cost of all products used in producing that unit (the first term on the right) plus value-added per unit produced. Just as the equations (14.2.1) provide the fundamental connection in multisectoral models between final demands and outputs, so these equations provide the fundamental connection between unit value added and prices. If we want to know how specific changes in productivity or in wages in one or several industries will affect prices in all industries, these equations are the key. If we calculate

the prices for v vector given in the table, we should find that all prices are equal to 1.

There is, furthermore, a relation of fundamental importance between the solutions of the two sets of equations. Namely, given any A, f, and v, the q and p which satisfy $q = Aq + f$ and $p = pA + v$ also satisfy

$$vq = pf \tag{14.2.3}$$

This equation says that the value of the final demands evaluated at the prices implied by equations (14.2.2) are equal to the payments to the resources necessary to produce those final demands by (14.2.1). Thus, if our outputs and prices satisfy the required equations, we can be certain that GDP measured by the final demands in current prices will be equal to the GDP measured by the payments to resources (or factors) in current prices. If we build these equations into our models, we can be certain that the models will satisfy the basic accounting identity in current prices. This relation may well be called the fundamental theorem of input-output analysis. Fortunately, it is as easy to prove as it is important, and you should produce your own proof. If you need help desperately, turn the book upside down and read it.

Multiply (14.2.1) on the left by p to get
(A) $pq = pAq + pf$
Multiply (14.2.2) on the right by q to get
(B) $pq = pAq + vq$
Subtract (B) from (A) to get
(C) $0 = pf - vq$ or $pf = vq$

14.3. Combining Input-Output and Institutional Accounts

The national accounts which we have presented so far in connection with the input-output table lack some of the concepts which we found very useful in macroeconomic modeling, such as Personal income, Personal disposable income, Personal saving, Personal income taxes, and Government transfers to persons. The basic "institutions" in national accounts are (1) Persons, (2) Businesses, (3) Governments, and (4) Rest of World. Sometimes businesses are divided between financial and non-financial businesses, but we will not make that distinction in *Tiny*. "Persons" includes non-profit corporations such as private universities. The Rest of the World, abbreviated as RoW, shows only transactions of "institutions" of other countries with the "institutions" of the country concerned.

Table 14.5. TINY: NIPA-Style Presentation
Year 2000

	Persons	
+	Labor income	1367
+	Interest and dividends received	220
+	Government transfers	150
=	Personal income	1737
-	Personal taxes	226
=	Disposable income	1511
-	Personal consumption expenditure	1477
=	Personal saving	34
	Business	
+	Depreciation	167
+	Capital income	259
-	Interest and dividends payed	220
-	Investment	224
=	Business saving	-28
	Governments	
+	Indirect taxes	215
+	Personal taxes	226
-	Government purchases of goods and services	-312
-	Government transfers to persons	-150
=	Government saving	-21
	Rest of World	
+	Imports	220
-	Exports	215
=	RoW saving	5

The institutional accounts begin with the allocation of components of value added from the input-output accounts to the institutions which receive them. Labor income is allocated to Persons; Depreciation and Capital income is allocate to Business; Indirect taxes are allocated to Governments. Government transfers, such as social insurance and welfare payments, are then moved from Governments to Persons, to give Personal income. Then taxes are moved from Persons and Business to Governments, with Disposable income as the balance.

There are several ways to present these accounts. The simplest is similar to that used in the USA NIPA and should be familiar from the discussion of the AMI model in Part 1 of this book.

A consequence of the fundamental identity of the total value added and the total final demand in the input-output table is that the total saving is identically zero. You can exercise your mental arithmetic to quickly verify this identity for *Tiny*. The NIPA-style account is clear, easy to read, and easy to convert into a program for calculation. Furthermore, data for several years can be conveniently shown in parallel columns that make comparison easy. Its disadvantage is that its form does not make evident why total saving is zero or what are matching entries. For example, the form of the accounts does not show that Personal taxes paid by Persons is the same as Personal taxes received by Governments.

That shortcoming is overcome in a second way of presenting the institutional accounts, a way I will call the Balances presentation. This presentation also makes clear why total saving is zero. It is shown in the table below.

Table 14.6. Institutional Accounts for TINY: Balances Presentation

Transaction	Persons	Business	Gov	RoW		PCE	Gov	Inv	NetExp
Primary distribution	1367	426	215	0	=	1477	312	224	-5
Interest and dividends	220	-220			=				
Gov't transfers	150		-150		=				
Balance: Inst. Income	1737	206	65		=	1477	312	224	-5
Direct taxes	-226		226		=				0
Balance: Disposable income	1511	206	291		=	1477	312	224	-5
Personal consumption	-1477				=	-1477			
Govenment purchases			-312		=		-312		
Business investment		-224			=			-224	
Net imports				5	=				5
Balance: Saving	34	-18	-21	5	=	0	0	0	0

In the first line, the "Primary distribution" of Value added, labor income is given to Persons; Depreciation and Capital income, to Business; and Indirect taxes, to Governments. To the right of the = sign are the components of Final demand. The sum of the items to the left of the = sign is, of course, equal to the sum of those on the right.

Next follow two transfer lines that (1) move Interest and dividends from the Business column to the Persons column, and (2) move Government transfers to persons from the Government column to the Persons column. The next line, labeled "Balance: Institutional Income," is a balance line, the sum of the preceeding lines. In the Persons column, it gives Personal income. Below it, the Direct taxes transfer line moves personal income taxes from Persons to Government and could also move corporate profit taxes from Business to Governments. (For *Tiny*, however, we have assumed that these corporate taxes are zero.) The next balance line, the sum of the previous balance line with the intervening transfer line, gives Disposable income by institution. Then follow the lines which subtract the final demand expenditures from the institutions which make them. The final balance line then gives the savings of each institution on the left of the = sign and zeroes on the right. Of course, the sum of the items on the left of this last line equals the sum of the items on the right, namely, zero. Thus, this presentation makes it clear why total saving, including that of the Rest of the World in our country, is always zero. The major disadvantage of this layout is that it cannot show data for several years in close proximity for easier comparison.

The international System of National Accounts (SNA) used by most countries other than the USA, uses a presentation based on the Balances Presentation, but somewhat more complicated and much less clear. Here it is for *Tiny*.

Table 14.7. Institutional Accounts for TINY: SNA-Style Presentation

Institution	Persons		Business		Governments		Rest of World	
Transaction	Sources	Uses	Sources	Uses	Sources	Uses	Sources	Uses
Primary distribution	1367		426		215		220	215
Interest and dividends	220			220				
Government transfers	150					150		
Personal tax		226			226			
Totals	1737	226	426	220	441	150	220	215
Balance:Disposable income	1511		206		291		5	
Personal consumption expenditures		1477						
Government expenditures						312		
Business investment				224				
Totals	1511	1477	206	224	291	312	5	
Balance: Saving	34		-18		-21		5	

Under each institution are two columns, one for sources of funds for the institutions and one for uses of funds. Instead of a single line for each of the balances, two lines are necessary, one to take the totals and one to show (in the Sources column) the result of subtracting total uses from total sources. I have not shown a balance line of Institutional of income (of which Personal income is a highly useful instance) because this concept plays no role in the SNA, which thus fail to give a concept useful as a base for calculating personal income taxes. The SNA presentation does not make clear why total saving is zero and requires two lines for each balance instead of one. However, I have seen a number of presentations in which the total lines ares omitted, thus making it very hard for the reader to figure out what is going on. The main virtue of the SNA presentation is that it largely avoids negative numbers.

Yet a fourth presentation combines the input-output table with the institutional accounts in what is called a Social Accounting Matrix or SAM. The SAM for *Tiny* is shown in Table 14.8. In an input-output table, the row sums equal the corresponding column sums for the industries. The SAM generalizes that idea so that all accounting identities are expressed by requiring the sum of each row to equal the sum of the corresponding column in a square matrix. In the SAM for *Tiny*, the first rows are those of the input-output table, both the products and the value-added. Below these rows, we add a row for each institution, one for each final demand column, and finally a row for saving. Between the columns for industries and the final demand columns we slip columns with the same names as the value-added rows, and then a column for each institution. After the final demand columns, we append one corresponding to the Savings row. The "Primary distribution" line of the SNA-Style accounts is then represented by the total of each type of value added into the cell at the intersection of row for the institution receiving the income and the column of the type of income. At this point, the row totals equal the column totals for the industries and for value-added components. The transfers among institutions are then shown by entering the amount in the row of the receiver and the column of the payer. The totals of each final demand column are entered into the corresponding row in the column of the institution purchasing that final demand. All row totals now equal corresponding column totals except for the four institutions. Their row totals are their receipts while their column totals are their expenditures. They differ by the amount of saving by each institution. So if we now enter these savings in the Saving row at the bottom of the table, the row totals equal the column totals also for the institutions. The row sum of the Saving row is, as has been said

repeatedly, zero, so to match the Saving row, we just need an all-zero Saving column.

Social Accounting Matrices have proven quite popular with economists. They are a way to combine national accounts with a consistent input-output table and institutional accounts. Their main advantage is that the form makes the consistency evident. But as the input-output table increases in detail, the SAM becomes worse as a way of actually viewing data. Consequently, we will make no further use of SAM's and will generally use the NIPA-like presentation because of the important advantage that data for several years can be shown in parallel columns.

To illustrate the use of integrated national accounts in combination with interindustry tables, we need historical series for at least the national accounts aggregates. I have made up such a data bank for *Tiny* with the values shown above for the year 2000 and with values for other years from 1978 to 2003 made up by assuming a movement similar to that of the corresponding entry in the USA NIPA. These "historical" series are in the *Tiny* data bank.

Table 14.8. A Social Accounting Matrix for TINY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	Ag	Min	G&E	Mfg	Com	Trans	Serv	Gov Ind	Dep	Labor	Capital	Ind Tax	Per-sons	Bus	Gov't	RoW	PCE	Gov	Invest	Exp	Imp	Sav	Tot
1 Ag	20	1	0	100	5	0	2	0									15	1	0	40	-20		164
2 Mining	4	3	20	15	2	1	2	0									2	1	0	10	-10		50
3 G&E	6	4	10	40	20	10	25	0									80	10	0	0	0		205
4 Mfg	20	10	4	60	25	18	20	0									400	80	200	120	-170		787
5 Commerce	2	1	1	10	2	3	6	0									350	10	6	10	0		401
6 Transport	2	1	5	17	3	2	5	0									130	20	8	5	0		198
7 Services	6	3	8	45	20	5	20	0									500	40	10	30	-20		667
8 GovInd	0	0	0	0	0	0	0	0									0	150	0	0	0		150
9 Deprec.	8	4	40	40	25	30	20	0															167
10 Labor	68	21	31	350	150	107	490	150															1367
11 Capital	20	2	66	60	40	12	59	0															259
12 IndTax	8	0	20	50	109	10	18	0															215
13 Persons										1367				220	150								1737
14 Firms									167		259												426
15 Gov't												215	226										441
16 RoW																							0
17 PCE													1477										1477
18 Gov Purch															312								312
19 Invest														224									224
20 Export																215							215
21 Import																-220							-220
22 Saving													34	-18	-21	5							0
23 Col Sum	164	50	205	787	401	198	667	150	167	1367	259	215	1737	426	441	0	1477	312	224	215	-220	0	

14.4. A Historical Note

All of us tend to presume that the world was made the way we found it; if there were input-output tables in it when we arrived, then they must have always been there. Of course, that is not the case. In fact, they are so much connected with the work of one man, Wassily W. Leontief, that without his remarkable contribution they would probably not have been developed until decades later. Born in St. Petersburg in 1906, he was already a university student when the Bolsheviks began taking over the educational program. He joined a group protesting this process, was caught pasting up a poster, spent a while in jail and was periodically jailed and interrogated thereafter. Though deeply interested in the economy of his country and in the efforts at economic planning, he clearly had little to hope for from the Bolshevik government. Even as an undergraduate, however, his paper on "The Balance of the Economy of the USSR" describing efforts in Russia to investigate interindustry relations came to the attention of professors in Germany. When he graduated from the University of Leningrad in 1926, he was offered the possibility of graduate study in Germany, but it was already difficult to get out of the Soviet Union. By an extraordinary turn of fate, he developed a bone tumor on his jaw. It was removed, but the surgeon warned him that he would surely soon die. Armed with the surgeon's written statement, he argued to the officials that he should be allowed to leave the country since he would certainly be useless and possibly expensive to the government. The argument worked, and in 1925 he arrived in Germany with the tumor in a bottle. It was there re-examined and found ... benign! His work in Germany led, via Nanjing, to an appointment at the National Bureau of Economic Research in New York. His theoretical writings came to the attention of the Harvard faculty which offered him an instructorship. He accepted the Harvard offer on the condition that he be given a research assistant to help him build what we would now call an input-output table. The reply informed him that the entire faculty had discussed his request and had unanimously agreed that what he proposed to do was impossible and, furthermore, that even if it were done, it would be useless. Nonetheless, they were so eager to have him come that they would grant the request and hope that he would use the resources for better purposes. He didn't. In 1936, his first results were published; in 1939 a book *Structure of the American Economy* appeared. It had input-output tables for the United States for 1919 and 1929. The theoretical parts of the book had the major ideas of input-output analysis: coefficients, simultaneous solution, and price equations. During World War II, Leontief constructed, with support of the U.S. Bureau of Labor Statistics (BLS), a 96-sector table for 1939 and, by 1944 was able to study changes in employment patterns which could be expected after the end of the war. In 1947, a second edition of the book appeared with the addition of a 1939 matrix and a comparison of input-output and single-equation projections.[3] In 1973, he was awarded the Nobel prize in economics for this work. Leontief remained active until shortly before his death in 1999 at the age of 93.

In 1949, a group at the BLS began work on a 400-sector table for 1947. A 190-sector table was published in 1952, but financing – which had come through the Defense budget – for the more than fifty people working on the project was discontinued early in the Eisenhower administration, so that neither the full table nor the extensive documentation of the details of its production were ever published.

---

3 The spelling of Leontief's name in Latin letters was for German speakers; English speakers almost invariably mispronounce it, though he never corrected anyone. In *Wassily,* the *W* is pronounced V, the *a* is long as in "father," and the accent is on the *si* which is pronounced "see". In *Leontief* the accent is on *on* and the *ie* is pronounced like the ye in "yet". The final *f* is a soft v.

In other countries, making of tables spread rapidly. They were incorporated in the United Nation's standard System of National Accounts prepared by Richard Stone. In 1950, the first international conference on input-output methods was sponsored by the United Nations; the eleventh (without U.N. support) was held in 1995.

In the late 1950's, Soviet authors, eager to make input-output acceptable in their country, put together a table for the Soviet Union in 1924 and argued that all the essential ideas had originated in the Soviet Union. The difference, however, between what they could find in the literature of that period and Leontief's comprehensive treatment only heightens an appreciation of his contribution.

Gradually, it has come to be recognized that an input-output table is not only useful for economic analysis and forecasting but is also an essential step in making reliable national accounts. The statistical offices of most major industrial countries, therefore, prepare input-output tables, often on a regular basis. Annual tables for France, the Netherlands, Norway, and Japan are prepared as a part of annual national accounting. In the USA, a comprehensive table is made every five years in the years of economic censuses (years ending in 2 and 7) and is used in revising and "benchmarking" the national accounts.

In 1988, the International Input-Output Association was organized as a group of individuals interested in using input-output techniques. In 1989, it began publishing its own journal, *Economic Systems Research*.

The Interdyme modeling system, like the *G7* program, was developed by the Inforum group in the Department of Economics at the University of Maryland. It has been used in developing and linking dynamic input-output models of about twenty countries. Most of these models have been developed and used mainly in the country concerned.

CHAPTER 15. INPUT-OUTPUT COMPUTATIONS

15.1. Introduction to Input-Output Computing with Just *G7*

In this section, we will see how to turn the *Tiny* input-output table and data bank into a simple input-output model using only commands available in *G7*. In this model, we will move each final demand column forward and backward over the period 1995 to 2003 by the index of the corresponding GDP component in the *Tiny* data bank. Then we move all the final demand vectors except investment up by 3.0 percent per year from 2003 to 2010. Investment is moved forward by a wavy series composed of a base series growing at 3.0 percent per year plus a sinusoidal function. Input-output coefficients and the composition of the five final demand components are kept constant. Outputs by each industrial sector are then calculated for every year from 1995 to 2010. With the additional assumption that the shares of each type of income in value added by each industry remain constant, we calculate income of each type in each industry. Piecewise linear trends in the input-output coefficients, value-added coefficients, and composition of the final demand vectors could easily be introduced, but that has been left as an exercise. This model is incomplete and somewhat inconsistent with itself for many reasons, including the following:

1. It does not assure consistency of Personal consumption expenditure with the Personal income it implies;
2. It does not relate the imports of a product to the domestic use of the product;
3. Investment is not detailed by industry and related to the growth of the industry as found by the model.

Introducing such features to exploit the full potential of input-out modeling will require the Interdyme software described in the following chapter. Despite these limitations, such simple models as the one described here, though with greater industry detail and more finely divided final demands, have been widely used by groups which have a macroeconomic model and want the industry outputs consistent with the its final demand forecasts.

Working with input-output in *G7* requires the use a VAM (Vectors And Matrices) file. As the name suggests, this type of data bank holds time series of vectors and matrices. *G7* has commands which can add, subtract, multiply, and invert matrices and add and subtract vectors and multiply them by matrices. Thus, the operations discussed so far, and several others, can easily be performed in *G7*. A VAM file differs in two important respects from the G data banks we have worked with so far:

1. In the standard G bank, all elements are the same size. Specifically, a time series of a single variable begins at the base year of the data bank and extends over the number of observations in the bank, as specified by the G.CFG file. In VAM files, elements are time series of vectors or matrices of various dimensions. As in the standard G bank, all time series are the same length.
2. In standard G banks, we can create new series as we work, for example, with *f*, *fex*, or *data* commands. In VAM files, we buy the flexibility of having elements of various sizes by specifying at the outset (in a file usually called VAM.CFG) the contents of the file, that is, the names and dimensions of each vector or matrix in the bank along with the names of the files giving the titles of the row or columns of the vector or matrix. One might suppose that it is a bit of nuisance to have to specify this structure of the VAM file at the outset. In practice, however, this need to pre-specify structure proves a useful discipline in building

complex models. If, as a model evolves, it becomes necessary to revise the specification of the VAM file, it is easy to copy the contents of the old file into the new, enlarged file, or simply to remake the VAM file.

We can illustrate the use of the VAM file and some new *G7* commands for making some simple calculations with the input-output table presented in section 1 of this chapter. In this example we will assume that the IO data are for the year 2000. The box below shows the VAM.CFG file for this model, which is called *Tiny* It and all the files used in this chapter are in TINY.ZIP. I suggest that you make a folder, copy TINY.ZIP into it, and unzip it.

Figure 15.1.

VAM.CFG for the *Tiny* Model

```
1995 2010
# Vam file for Simplest Model
FM          8   8  0  sectors.ttl sectors.ttl #Input-output flow matrix
AM          8   8  0  sectors.ttl sectors.ttl #Input-output coefficient matrix
LINV        8   8  0  sectors.ttl sectors.ttl # Leontief inverse
out         8   1  3  sectors.ttl # Output
pce         8   1  0  sectors.ttl # Personal consumption expenditure
gov         8   1  0  sectors.ttl # Government spending
inv         8   1  0  sectors.ttl # Investment
ex          8   1  0  sectors.ttl # Exports
im          8   1  0  sectors.ttl # Imports
fd          8   1  0  sectors.ttl # Total final demand
dep         8   1  0  sectors.ttl # Depreciation
lab         8   1  0  sectors.ttl # Labor income
cap         8   1  0  sectors.ttl # Capital income
ind         8   1  0  sectors.ttl # Indirect taxes
depc        8   1  0  sectors.ttl # Depreciation coefficients
labc        8   1  0  sectors.ttl # Labor income coefficients
capc        8   1  0  sectors.ttl # Capital income coefficients
indc        8   1  0  sectors.ttl # Indirect taxes coefficients
pcec        8   1  0  sectors.ttl # Personal consumption shares
invc        8   1  0  sectors.ttl # Investment shares
govc        8   1  0  sectors.ttl # Gov shares
exc         8   1  0  sectors.ttl # Export shares
imc         8   1  0  sectors.ttl # Import shares
x           8   1  0  sectors.ttl # Working space
y           8   1  0  sectors.ttl # Working space
```

The first line in VAM.CFG gives the beginning and ending years for the VAM file. The next line, the one beginning with a #, is a comment to clarify the structure of the file. Comments beginning with a # can be placed anywhere in the file. Next are the lines describing the vectors and matrices. Each line shows the following:

1. The name of the vector or matrix.
2. Its number of rows.
3. Its number of columns.
4. The maximum number of lags with which a vector occurs in the model or a 'p' if the matrix is a “packed matrix” – a device useful in large-scale models.
5. The name of a file containing the rows titles of a vector or matrix.

6. If applicable, the name of a file containing the columns titles of a matrix.
7. A # followed by a brief description of the element.

As far as the computer is concerned, these lines are free format; all that is needed is one or more spaces between each item on a line. However, this is a file also read by humans, so putting in spaces to make the items line up in neat columns is a good idea. Figure 15.1 shows the VAM.CFG file for the *Tiny* model based on example of section 1 of the last chapter. (The VAM.CFG file on the zip file has more vectors than shown here. The extra ones will be used in the next chapter and, in the meanwhile, will do no harm.)

To create a vam file from a vam configuration file the command in *G7* is

```
vamcreate  <vam configuration file> <vam file>
```

For example, to create the vam file HIST.VAM from the configuration file VAM.CFG, the command is

```
vamcreate vam.cfg hist
```

The "vamcreate" command may be abbreviated to "vamcr", thus:

```
vamcr vam.cfg hist
```

At this point, the newly created vam file has zeroes for all its data. We will now see how to populate the bank and work with the data. The first step is to assign it as a bank. The command is

```
vam  <filename> <letter name of bank>
```

For example, we could assign HIST.VAM to the 'b' position by typing:

```
vam hist b
```

Letters 'a' through 'v' may be used to designate banks. However, it is generally a good practice to leave 'a' as the G bank which was initially assigned.

In order to avoid continually entering the bank letter, most commands for working with VAM files use the default VAM file. It is specified by the "dvam" command

```
dvam <letter name of bank>
```

For example, we can set the VAM file in position 'b' as the default by typing:

```
dvam b
```

A vam file must already be assigned as a bank before it can be made the default. However, if several VAM files are assigned, the default can be switched from one to another as often as needed.

The usual ways to introduce data into a VAM file are with the "matin" command for matrices and the "vmatdat" command for vectors. We can illustrate them with the data for *Tiny* from section 14.1.

Figure 15.2

The Flows.dat File for Introducing the Input-Output Flow Matrix into the VAM File

```
matin FM 2000 1 8 1 8  15
#            Agricul Mining Elect  Mfg Commerce Transp Services Govt
Agriculture       20      1     0  100        5      0        2    0
Mining             4      3    20   15        2      1        2    0
Electricity        6      4    10   40       20     10       25    0
Manufacturing     20     10     4   60       25     18       20    0
Commerce           2      1     1   10        2      3        6    0
Transportation     2      1     5   17        3      2        5    0
Services           6      3     8   45       20      5       20    0
Government         0      0     0    0        0      0        0    0
```

The “matin” command on the first line is followed by the matrix name in VAM.CFG, then by the year to which the matrix belongs, then the number of the first row and last row in the following rectangle of data, then the number of the first column and last column in the rectangle. (In the present case, the rectangle is the whole table; but this ability to read in a table rectangle-by-rectangle is quite useful for reading tables scanned from printed pages.) The last number on the “matin” line is the skip count, which specifies the number of characters to be skipped at the beginning of each line. These characters usually give sector names or numbers. The # in the first position marks the second line as a comment. Then come the data; each line is in free format after the initial skip. (Do not use tabs in characters which are to be skipped; the tab character will be counted as just one character.)

The FD.dat file shown below in Figure 15.3 illustrates the introduction of vectors, in this case, the final demands. The “vmatdat” command is rather flexible; it can introduce a number of vectors for one year or one vector for a number of years. The vectors can be the rows or the columns in the following rectangle of data. Because of this flexibility, we have to tell the command how to interpret the rectangle of data. The command must therefore by followed by a 'c' or an 'r' to indicate whether the vectors appear as columns or rows in the following rectangle of data. Here, the vectors are clearly columns. The next number is the number of vectors in the rectangle; here 5. Next is the number of years represented in the rectangle. Here it is 1, for the columns are different vectors for the same year. (Either the number of vectors or the number of years must be 1.) The next two numbers are the first and last element numbers of the data in the rectangle, and the last is the skip count. Since this command is introducing several vectors for one year, that year is specified at the beginning of the next line, and the names of the vectors follow it. (If we were introducing data for one vector for several years, the vector name would be in the first position on this line, followed by the year numbers.)

Figure 15.3. The FD.DAT File for Introducting the Final Demands into the VAM File

```
vmatdata c 5 1 1 8 15
2000            pce  gov   inv     ex      im
#            PersCon   Gov Invest Exports
Imports
Agriculture      15     1     0    40     -20
Mining            2     1     0    10     -10
Electricity      80    10     0     0       0
Manufacturing   400    80   200   120    -170
Commerce        350    10     6    10       0
Transportation  130    20     8     5       0
Services        500    40    10    30     -20
Government        0   150     0     0       0
```

The value-added rows are introduced by the "vmatdat" command and data shown in the box below. In this example, vectors are read in as rows.

Figure 15.4

The VA.DAT File for Introducing the Value-added Vectors

```
vmatdata r 4 1  1 8 15
2000  dep lab cap ind
#                1      2      3      4      5      6      7      8
Depreciation     9      4     40     40     25     30     20      0
Labor           68     21     31    350    150    107    490    150
Capital         20      2     56     60     40     12     59      0
Indirect tax     8      0     20     50    109     10     18      0
```

Here, finally, are the *G7* commands to create the VAM file and load the data into it:

```
# tiny.pre – Create the VAM file for Tiny
vamcreate vam.cfg hist
vam hist b
dvam b
# Bring in the intermediate flow matrix
add flows.dat
# Bring in the final demand vectors
add fd.dat
# Bring in the value added vectors
add va.dat
```

The complete set of commands for making the calculations described in this section are in the file GMODEL.PRE, shown in figure 15.5. To fit this large file on a single page, some commands have been doubled up on a single line but separated by a semicolon – a trick which works in *G7* just as in C++.

Figure 15.5. Gmodel.pre File to Build a *Tiny* Model Using only *G7*, No Interdyme

```
zap; clear
bank tiny
vamcreate vam.cfg hist
vam hist b; dvam b
# Bring in the intermediate flow matrix
add flows.dat
show b.FM y 2000
# Bring in the final demand vectors
add fd.dat
# Bring in the value added vectors
add va.dat
fdates 2000 2000
# Add up the intermediate rows
getsum FM r out
# Add on the final demand vectors to get total output
vc out = out+pce+gov+inv+ex+im
show b.out
# Copy intermediate flows to AM and convert to coefficients
mcopy b.AM b.FM
coef AM out
vc depc = dep/out; vc labc = lab/out
vc capc = cap/out; vc indc = ind/out
# Copy the 2000 coefficient matrices to all the other years
fdates 1995 2010
# Copy the 2000 AM matrix into 1995 - 2010
dfreq 1
f one = 1.
index 2000 one AM
# Demonstrate that AM has been copied by showing its first column.
show b.AM c 1
index 2000 one depc; index 2000 one labc
index 2000 one capc; index 2000 one indc
# Move the four final demand columns by their totals
# in the historical years, 1995 - 2003
fdates 1995 2003
index 2000 pcetot pce; index 2000 invtot inv ; index 2000 govtot gov
index 2000 extot  ex; index 2000 imtot  im
# Extend the final demands from 2003 to 2010 using a
# 3 percent growth rate for all but inv and a wavy
# pattern for it.
fdates 1995 2010
# Create a time trend
f time = @cum(time,one,0)
f g03 = @exp(.03*(time-9))
ty g03
f waves = g03 + .3*@sin(time-9)
ty waves
fdates 2003 2010
index 2003 g03   pce; index 2003 waves inv; index 2003 g03   gov
index 2003 g03   ex; index 2003 g03   im
# Take the Leontief inverse of the A matrix
fdates 1995 2010
mcopy b.LINV b.AM
linv LINV
show b.LINV y 2000
# Add up the final demands
vc fd = pce+gov+inv+ex+im
show b.fd
# Compute total outputs
vc out = LINV*fd
show b.out
# Compute Value added
# The following are element-by-element multiplication
vc dep = depc*out; vc lab = labc*out
vc cap = capc*out; vc ind = indc*out
gdates 1995 2003 2010
fadd graphs.fad sectors.ttl
```

Now let us look at some of the data we have introduced by displaying them in a grid on the screen. The command

```
show FM   y   2000
```

will show a spreadsheet-like grid containing the flow matrix (FM) for the year 2000. To adjust the default column width and the number of decimal places in the display, click the Options menu item. Not only does this display look like a spreadsheet, it also works like one in that you can copy and paste data from one to the other.

To look at a row, say row 2, of the FM matrix for all years of the VAM file, the command is

```
show FM r 2
```

Similarly, to show column 5 for all years, the command is:

```
show FM c 5
```

Thus, in showing a matrix, we have to choose among showing the whole matrix for one year and showing one row or column for all years. The choice is indicated by the letter – a 'y', 'r' or 'c' – following the matrix name.

Showing vectors is simpler because we do not have to make this choice; we just name the vector and get all values for all years. Here are two examples

```
show ind     # Display the indirect tax vector
show b.pce   # Display the personal consumption expenditure vector
```

The second of these examples illustrates that the "show" command allows us to specify a bank other than the default VAM file.

Now that we have added the data and displayed it to check that it was accurately read, we can begin to perform computations. To calculate the input-output coefficient matrix, we need out, the vector of outputs by industry. It was not read in, but it can be computed by summing the rows of the FM matrix and then adding to this row sum the final demand columns. Here are the two calculations and the "show" command to view the result:

```
# Add up the intermediate rows
getsum FM r out
# Add on the final demand vectors to get total output
vc out = out+pce+gov+inv+ex+im
show b.out
```

We are now ready to copy the flow matrix, stored in FM, to AM and then convert it to input-output coefficients by dividing each element of each column by the corresponding element of the out vector. We copy the matrix with the "mcopy" command. The general form of the "mcopy" command to copy matrix or vector A from bank x to element B in bank y is

```
mcopy  y.B [=] x.A
```

The = sign is optional but is useful reminder of which way the copy is going. The y. is optional if y is the default VAM file, and the same is true for the x.. Since this copy and these calculations need be done only for one year, 2000, we first set the fdates so that the "mcopy" and "coef" commands work only on the years from 2000 to 2000 (which is to say, only for 2000). Here are the commands

```
# Copy intermediate flows to AM and convert to coefficients
fdates 2000 2000
mcopy b.AM = b.FM
coef AM out
show AM y 2000
# Create value-added coefficient vectors.
vc depc = dep/out
vc labc = lab/out
vc capc = cap/out
vc indc = ind/out
# Set fdates back to the entire range of the VAM file.
fdates 1995 2010
```

With the input-output coefficients calculated, we can now go on to illustrate finding the Leontief inverse, calculating outputs from exogenous forecasts of final demands, calculating value-added components, and displaying, graphing, and making tables of the results. We will first copy the input-output coefficient matrix and the value-added coefficient vectors from 1995 to 2010. We can conveniently do this with *G7*'s "index" command. This command is used to move all elements of a vector or matrix in the default VAM file forward or backward in proportion to a guide series. Its general form is:

```
index <base year>  <guide series>  <matrix or vector>
```

It operates over the range specified by the current fdates. Since we just want to copy the coefficients to all the years, our guide series will be simply a series of 1's, which we shall call *one*. Here are the commands

```
# Copy the 2000 AM matrix into 1995 - 2010
dfreq 1
f one = 1.
index 2000 one AM
index 2000 one depc
index 2000 one labc
index 2000 one capc
index 2000 one indc
show AM c 1
```

The last command displays the first column of the AM matrix for all the years in a grid; all columns of this display should, of course, be identical. For purposes of illustration, we will let *AM* remain constant in all years.

The final demands, however, will move in a slightly more interesting way. Between 1995 and 2003, the elements of each final demand column will follow the index of the total of that column as given by the corresponding aggregate in the national accounts. Here are the *G7* commands to make that happen.

```
# Move the four final demand columns from the 2000 value by their totals
```

```
# in the historical years, 1995 - 2003
fdates 1995 2003
index 2000 pcetot pce
index 2000 invtot inv
index 2000 govtot gov
index 2000 extot  ex
index 2000 imtot  im
```

From the base of 2003, we will have all components of final demand except investment grow at a steady 3 percent per year to 2010. Investment will also have one component growing at this same rate but added to it – to make the results more interesting to view – will be a sine curve with a period of 2π years. Here are the commands for this operation.

```
fdates 1995 2010
# Create a time trend
f time = @cum(time,one,0)
f g03 = @exp(.03*(time-9))
f waves = g03 + .3*@sin(time-9)
fdates 2003 2010
index 2003 g03   pce
index 2003 waves inv
index 2003 g03   gov
index 2003 g03   ex
index 2003 g03   im
```

To add up the components of final demand to *fd*, we use the "vc" (for vector calculation) command. It can add up any number of vectors to get a total. Here are the commands.

```
# Add up the final demands
vc fd = pce+gov+inv+ex+im
show fd
```

We are now going to ignore the fact that the AM matrix is the same in all years – we could have changed it had we wanted to – and take its Leontief inverse in all years in the fdates range. The command

```
linv  <square matrix> [year]
```

converts the square matrix into its Leontief inverse. For example,

```
linv A
```

converts A into $(I-A)^{-1}$. We then multiply this inverse by the final demand vector to compute the output vector. The "linv" command works over the fdate range unless the optional year argument is present.

```
# Take the Leontief inverse of the A matrix
mcopy LINV = AM
linv LINV
show LINV y 2000
```

```
# Compute total outputs
vc out = LINV*fd
show b.out
```

With the outputs known, we can compute the implied value-added of each type by each industry with the following commands. In them, the "vc" command will recognize that the dimensions of the vectors on the right are such that element-by-element multiplication makes sense and perform the calculation.

```
# Compute Value added
# The following are element-by-element multiplication
vc dep = depc*out
vc lab = labc*out
vc cap = capc*out
vc ind = indc*out
show lab
```

As we went along, we showed results in spreadsheet-like grids to check that our answers were generally reasonable. Now we need to graph the results. In doing so, we use the fact that elements of vectors in a VAM file can be referred to in *G7* simply by the name of the vector followed by a numeral. We can graph the second element of the out and pce vectors from the VAM file assigned as bank 'b' with the graph command like this:

```
gr  b.out2   b.pce2
```

If the VAM file is the default, we can omit the bank letter and period. Thus, in the instance just given, we could just use the following command:

```
gr out2  pce2
```

This way of referring to a time series of elements of a vector works also for "type" and "r" commands and for the right-hand side of "f" or "fex" commands. Similarly, we can refer to an element of a matrix in a type, graph, or regression command or the right side of an "f" command. Specifically, to retrieve an element of a matrix, type the matrix name followed by the row number, followed by a dot, followed by the column number. For example,

```
type AM3.5
```

will print to the screen the values of the element in the third row and fifth column of the AM matrix.

We can get a lot more graphs very quickly by use of *G7*'s "fadd" command. The name "fadd" is a contraction of "file-directed add command." It works with text substitution in a way that is very convenient in working with multisectoral models. The general form is

```
fadd  <command file>   <argument file>
```

In our case, the "command file" will be GRAPHS.FAD:

```
vr 0
ti %3 %5
subti Output and Final demand
gname out%3
gr b.out%3 b.fd%3
```

```
subti Depreciation,Labor income, Capital income, Indirect taxes
gname va%3
gr b.dep%3 b.lab%3 b.cap%3 b.ind%3
ti
subti
```

The argument file will be the same SECTORS.TTL file which we used for supplying row and column titles for the matrices and vectors in the VAM file, namely:

```
Agricul     ;1   e   "Agriculture"
Mining      ;2   e   "Mining and quarrying"
Elect       ;3   e   "Electricity and gas"
Mfg         ;4   e   "Manufacturing"
Commerce    ;5   e   "Commerce"
Transport   ;6   e   "Transportation"
Services    ;7   e   "Services"
Government  ;8   e   "Government"
```

Note that some of the lines in the command file – for example, the second – have a % followed by a number. These numbers refer to "arguments" from the "argument" file. For example, on the first line of the argument file, argument 1 is Agricul, argument 2 is ;, argument 3 is 1, argument 4 is e , and argument 5 is Agriculture . Normally an argument is delimited by a space or punctuation. Enclose arguments which contain spaces – such as the names of some sectors – in quotation marks. When the second line of the command file,

```
ti  %3  %5
```

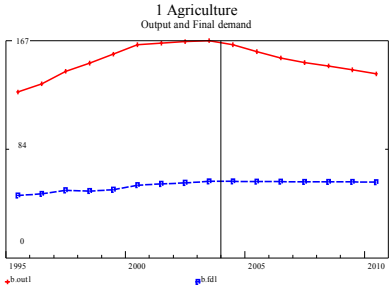
is executed with the arguments 3 and 5 from the first line of the argument file replacing the %3 and %5, the effect is that *G7* executes the command

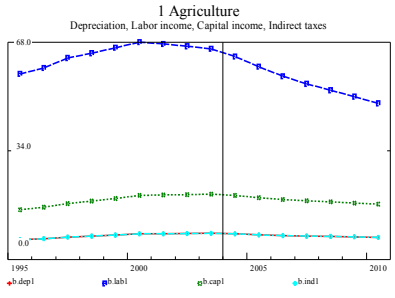
```
ti 1 Agriculture
```

The effect of the "fadd" command is that the entire command file is executed first with arguments from the first line of the argument file, then with the arguments from the second line of the argument file, and so on. Thus, with the single command

```
fadd  graphs.fad  sectors.ttl
```

G7 will draw graphs like the two shown below for Agriculture for all sectors.





We have used some but not all of the *G7* commands for matrix arithmetic in a VAM file. For reference, here are some others.

minv A	converts A into its inverse
madd A = B + C	adds B and C and stores in A
madd A = B - C	subtracts C from B and stores result in A
mmult A = B*C	multiply B and C and store result in A
mmult A = B'C	multiplies B transpose by C and stores result in A
mmult A = B&C	does element-by-element multiplication of B and C and stores in A
mmult A = B/C	element-by-element division of B by C stored in A
mtrans A B	the transpose of B is stored in A

In all of them, the command may be followed by an optional year in which to do the calculation; absent the year, the calculation is done for all years in the *fdates* range.

For tabulating the contents of a VAM file, we use exactly the same program, *Compare*, as we have used for macro models. It has, however, some features used exclusively with vectors and matrices. First of all, when we click Model | Tables on the *G7* main menu, we need to choose "vam" as the type of the first bank, then give "hist" as its name; in the "Stub file" control, fill in "tiny", and in the "Output file name" box type "tiny.out".

Figure 15.6

The TINY.STB File

```
\dates 1995 2000 2005 2010 1995-2000 2000-2005 2005-2010
\pages off
\noformat
\title TINY G-ONLY MODEL, ILLUSTRATIVE FORECAST

; out  Output of Industries
&
out1 ;1 Agriculture
out2 ;2 Mining and quarrying
out3 ;3 Electricity and gas
out4 ;4 Manufacturing
out5 ;5 Commerce
out6 ;6 Transportation
out7 ;7 Services
out8 ;8 Government
;
\add tiny.tab pce "Personal Consumption Expenditure"
;
\add tiny.tab gov "Government Expenditures"
;
\add tiny.tab inv "Investment by Supplying Industry"
;

# The next line forces a new page
*
\matcfg Matlist.cfg
\center Matrix Listing
\row
\cutoff .001
\matlist 1-8
```

The first line with the "\dates" command is familiar from macro models. Since we want to bring the

results into a word processor for printing, I have turned off the page numbering and all commands to the printer in the next two lines. The "\title" command gives a title to be printed across the top of each page of output. As with macro stub files, a line beginning with a ";" just puts the rest of the line in the output file, and a "&" command puts a line of dates across the page. The next eight lines print the output and its growth rates for the eight industries of the *Tiny* model for the dates specified.

We have not previously used *Compare's* "\add" command, which works just like *G7*'s "add" command, including a feature of the "add" command which we have yet used, namely, that it accepts arguments. The TINY.TAB file is shown in the box below. Instead of the lines in TINY.STB for printing the output of industries, we could have used the single line

\add tiny.tab out "Output of Industries"

The effect would have been exactly the same.

Figure 15.7

The TINY.TAB File

```
; %1 %2
&
%11 ;1 Agriculture
%12 ;2 Mining and quarrying
%13 ;3 Electricity and gas
%14 ;4 Manufacturing
%15 ;5 Commerce
%16 ;6 Transportation
%17 ;7 Services
%18 ;8 Government
```

The TINY.TAB is a bit confusing because of the strings "%11" , "%12", and similar strings below them. To the eye, this may look like a reference to argument 11 or argument 12. But the computer knows that there can be only nine arguments and thus the third character in these strings is not part of the argument specification. It will read these as "argument 1 followed by the character 1" or "argument 1 followed by the character 2."

The results the tabulations described thus far are shown in Figure 15.8 below.

The last five lines of TINY.STB are concerned with making a matrix listing from the VAM file. A matrix listing is best explained by looking at the results, which are shown for the first three industries in the Figure 15.9 below. For each row of the input-output table, the matrix listing shows each element of the identity:

output = intermediate demand + final demand.

Indeed, each element is shown for each year specified by the "\dates" command. Growth rates of the element are shown for the periods specified by the same command. This matrix listing technique is important not only for the information it displays, but also the consistency of the forecasts which it emphasizes.

Figure 15.8

TINY G-ONLY MODEL, ILLUSTRATIVE FORECAST

out Output of Industries

	1995	2000	2005	2010	95-00	00-05	05-10
1 Agriculture	140.7	164.0	189.6	216.7	3.1	2.9	2.7
2 Mining and quarrying	43.5	50.0	57.5	66.1	2.8	2.8	2.8
3 Electricity and gas	171.1	205.0	228.6	263.8	3.6	2.2	2.9
4 Manufacturing	663.0	787.0	908.3	1030.9	3.4	2.9	2.5
5 Commerce	331.0	401.0	439.9	510.0	3.8	1.9	3.0
6 Transportation	164.3	198.0	220.3	254.4	3.7	2.1	2.9
7 Services	555.8	667.0	738.2	854.7	3.6	2.0	2.9
8 Government	133.2	150.0	177.7	206.5	2.4	3.4	3.0

pce Personal Consumption Expenditure

	1995	2000	2005	2010	95-00	00-05	05-10
1 Agriculture	12.4	15.0	16.3	19.0	3.9	1.7	3.0
2 Mining and quarrying	1.6	2.0	2.2	2.5	3.9	1.7	3.0
3 Electricity and gas	65.9	80.0	87.1	101.2	3.9	1.7	3.0
4 Manufacturing	329.7	400.0	435.7	506.2	3.9	1.7	3.0
5 Commerce	288.5	350.0	381.2	442.9	3.9	1.7	3.0
6 Transportation	107.2	130.0	141.6	164.5	3.9	1.7	3.0
7 Services	412.1	500.0	544.6	632.7	3.9	1.7	3.0
8 Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0

gov Government Expenditures

	1995	2000	2005	2010	95-00	00-05	05-10
1 Agriculture	0.9	1.0	1.2	1.4	2.4	3.4	3.0
2 Mining and quarrying	0.9	1.0	1.2	1.4	2.4	3.4	3.0
3 Electricity and gas	8.9	10.0	11.8	13.8	2.4	3.4	3.0
4 Manufacturing	71.0	80.0	94.8	110.1	2.4	3.4	3.0
5 Commerce	8.9	10.0	11.8	13.8	2.4	3.4	3.0
6 Transportation	17.8	20.0	23.7	27.5	2.4	3.4	3.0
7 Services	35.5	40.0	47.4	55.1	2.4	3.4	3.0
8 Government	133.2	150.0	177.7	206.5	2.4	3.4	3.0

inv Investment by supplying industry

	1995	2000	2005	2010	95-00	00-05	05-10
1 Agriculture	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2 Mining and quarrying	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3 Electricity and gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4 Manufacturing	147.2	200.0	240.2	257.5	6.1	3.7	1.4
5 Commerce	4.4	6.0	7.2	7.7	6.1	3.7	1.4
6 Transportation	5.9	8.0	9.6	10.3	6.1	3.7	1.4
7 Services	7.4	10.0	12.0	12.9	6.1	3.7	1.4
8 Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Figure 15.9

TINY G-ONLY MODEL, ILLUSTRATIVE FORECAST

Matrix Listing

Seller: 1 Agriculture

	1995	2000	2005	2010	95-00	00-05	05-10
Sales to Intermediate							
1 Agriculture	17.2	20.0	23.1	26.4	3.1	2.9	2.7
2 Mining and quarrying	0.9	1.0	1.2	1.3	2.8	2.8	2.8
4 Manufacturing	84.2	100.0	115.4	131.0	3.4	2.9	2.5
5 Commerce	4.1	5.0	5.5	6.4	3.8	1.9	3.0
7 Services	1.7	2.0	2.2	2.6	3.6	2.0	2.9
SUM: Intermediate	108.1	128.0	147.4	167.7	3.4	2.8	2.6
Sales to Other Final Demand							
Personal consumption expenditure	12.4	15.0	16.3	19.0	3.9	1.7	3.0
Government consumption	0.9	1.0	1.2	1.4	2.4	3.4	3.0
Exports	33.1	40.0	45.4	52.8	3.8	2.5	3.0
Imports	-13.7	-20.0	-20.8	-24.1	7.6	0.8	3.0
Output	140.7	164.0	189.6	216.7	3.1	2.9	2.7

Seller: 2 Mining and quarrying

	1995	2000	2005	2010	95-00	00-05	05-10
Sales to Intermediate							
1 Agriculture	3.4	4.0	4.6	5.3	3.1	2.9	2.7
2 Mining and quarrying	2.6	3.0	3.5	4.0	2.8	2.8	2.8
3 Electricity and gas	16.7	20.0	22.3	25.7	3.6	2.2	2.9
4 Manufacturing	12.6	15.0	17.3	19.6	3.4	2.9	2.5
5 Commerce	1.7	2.0	2.2	2.5	3.8	1.9	3.0
6 Transportation	0.8	1.0	1.1	1.3	3.7	2.1	2.9
7 Services	1.7	2.0	2.2	2.6	3.6	2.0	2.9
SUM: Intermediate	39.5	47.0	53.2	61.0	3.5	2.5	2.7
Sales to Other Final Demand							
Personal consumption expenditure	1.6	2.0	2.2	2.5	3.9	1.7	3.0
Government consumption	0.9	1.0	1.2	1.4	2.4	3.4	3.0
Exports	8.3	10.0	11.4	13.2	3.8	2.5	3.0
Imports	-6.8	-10.0	-10.4	-12.1	7.6	0.8	3.0
Output	43.5	50.0	57.5	66.1	2.8	2.8	2.8

Seller: 3 Electricity and gas

	1995	2000	2005	2010	95-00	00-05	05-10
Sales to Intermediate							
1 Agriculture	5.1	6.0	6.9	7.9	3.1	2.9	2.7
2 Mining and quarrying	3.5	4.0	4.6	5.3	2.8	2.8	2.8
3 Electricity and gas	8.3	10.0	11.1	12.9	3.6	2.2	2.9
4 Manufacturing	33.7	40.0	46.2	52.4	3.4	2.9	2.5
5 Commerce	16.5	20.0	21.9	25.4	3.8	1.9	3.0
6 Transportation	8.3	10.0	11.1	12.8	3.7	2.1	2.9
7 Services	20.8	25.0	27.7	32.0	3.6	2.0	2.9
SUM: Intermediate	96.3	115.0	129.6	148.8	3.5	2.4	2.8
Sales to Other Final Demand							
Personal consumption expenditure	65.9	80.0	87.1	101.2	3.9	1.7	3.0
Government consumption	8.9	10.0	11.8	13.8	2.4	3.4	3.0

Perhaps you are wondering how the *Compare* program knows what elements go into the identity and what are the names of the sectors and final demands. The answer was given to the program in the "matrix listing configuration file" whose name, MATLIST.CFG, was given to *Compare* in the command

\matcfg matlist.cfg

The matrix listing configuration file to produce the matrix listing shown above is in the box below.

The MATLIST.CFG File for TINY

```
Matrix listing identity;out=AM*out+pce+gov+inv+ex+im
# Title file name for the rows of out, the lefthand side vector
out; "sectors.ttl"
# Title file names for matrix columns
AM; "sectors.ttl"
# headers for each term
header for out;    "Output"
header for AM*out; "Intermediate"
header for pce;    "Personal consumption expenditure"
header for gov;    "Government consumption"
header for inv;    "Investment"
header for ex;     "Exports"
header for im;     "Imports"
```

In the MATLIST.CFG file, any line beginning with a # is a comment and anything before the " ; " is likewise a comment. The first line gives the crucial identity on which the matrix listing is built. Recall that *Compare* has the VAM file and thus knows all the matrix and vector names and dimensions. It knows how to correctly interpret the expression "AM*out." The next line gives the file name of the sector titles for the vector on the left. The following line provides the file names for column titles of any matrices appearing in the identity. Here we have only one such matrix. Next come headers for each section of the table, where a section is a vector or a matrix-vector product.

We return now to the TINY.STB file to explain the last four lines, namely

\center Matrix Listing
\row
\cutoff .001
\matlist 1-8

The "\center" command centers text on the page. The command "\row" tells Compare to interpret the identity of the matlist configuration file as an identity in the rows. (The other possibility, "\column", would be used for showing the identity – that holds only in current prices – between the value of the output of an industry and the sum of its intermediate inputs and value-added components.) The "\cutoff" command eliminates the printing of entries which account for less than the specified fraction of the total of the row or column being listed.

Finally, the "\matlist" command instructs Compare to make the matrix listing for the group of sectors following the command. This is our first encounter with the *group* concept which is quite useful when working with multisectoral models. A group is just a collection of integers; it can be specified in a rather flexible way. Our specification, 1 - 8, means every sector from 1 to 8. An equivalent specification would be 1 2 3 4 5 6 7 8. If we want just 1 to 3 and 6 to 8, we could write any of following:

1-3 6-8
1 2 3 6 7 8
1-8 (4 5)

The numbers in the parenthesis are omitted from the list created by the ranges to the left; the parenthesis can also include ranges.

You may now want to ask, "Shouldn't we connect personal consumption expenditure to labor and capital income?" Of course we should, but to do so goes beyond what we can do in *G7* alone. It requires the Interdyme modeling system, which is similar to *Build* but for multisectoral models. Everything we have covered in this section is directly relevant to working with Interdyme, but surely you need to pause here and be sure that you have mastered the large amount of information we have already covered.

What better exercise could there be than to build your own *Tiny* model? Please complete exercise 1 and the others to explore some other ideas.

Exercises

1. Make up the input-output table for your own imaginary economy in 2005. It should have five to ten sectors, but not eight. Use different sector titles. Make forecasts to 2025. Graph the forecasts and make tables of output and other vectors. Make a matrix listing.
2. For the economy of our example, what levels of output and use of primary imputs would be required for the final demand (40, 6, 100, 600, 400, 170, 700, 148)?
3. How much of each of the four factors does one dollar of each of the final demands contain?
4. Was this economy a net exporter or importer of depreciation?
5. What would happen to the prices of each of the eight products if all indirect taxes were eliminated?
6. Greenhouse gases are emitted by the production of the various sectors of our model economy. Measured in tons per billion dollars of output, the emission coefficients for the various sectors of our economy are

   2.1 1.3 6.1 1.8 1.0 4.3 0.8 0.0

   What is the emission of greenhouse gases per billion dollars of final demand for each of the eight products? How much is attributable to a billion dollars of each of the types of final demand -- consumption, government, etc.? Was this country a net exporter or importer of greenhouse gas emissions?
7. The input-output flow table illustrated in the text was for year A. A comparable table for the same country but for a later year, year B, may be found in the files YBF.DAT, YBX.DAT and YBV.DAT in the TINY.ZIP file. (You have to fix up the correct commands to get the data into G.) Price indexes for the eight sectors from year A to B are given by the vector

   (1.01 1.10 1.06 1.07 1.15 1.24 1.18 1.20),

   while the cost of labor increased twenty percent between the two years. (The price indexes are in the file PINDEX.DAT.) What has happened between the two years to total labor requirements for producing one unit of final demand for each product?

8. Return to exercise 7 but now consider that the depreciation and capital income are produced with material inputs in the proportions given by the investment vector of the year in question. Ignore the indirect taxes and imports. The reciprocals of the labor requirements are productivity indexes for the economy in producing the various products supplied to final demand.

   Exercises 7 and 8 illustrate correct ways of studying productivity of the economy in making various final products. As we noted in section 14.1, it is impossible to know what has happened to productivity in a single *industry*, because the industry may have reduced its primary inputs while increasing its intermediate inputs; and the double-deflation method, supposed to handle this problem, is totally fallacious. The same problem does not arise in looking at total labor required, indirectly as well as directly, for the production of each unit delivered to final demand, for if the direct supplier to final demand has shifted required labor to other industries by buying more intermediate goods, that indirect labor will be automatically picked up. Thus, input-output calculations may offer a way of studying trends in productivity by product which elude methods which do not take into account indirect effects.

9. Read the *G7* help file for the "lint" command. Specify different (but consistent) values of the AM matrix, value-added coefficient vectors, and final demand vector shares for 1995 and 2010, use the "lint" command to interpolate values for other years, and repeat the calculations of the text with these time-varying coefficients. *Consistent* here means that the final-demand share columns sum to 1.0 and the sum of each column of AM plus the sum of the value-added shares in the same industry equals 1.0. Thus, these calculations are, in essence, in current prices, not constant prices.

15.2. Iterative Solutions of Input-output Equations

Before moving on to the Interdyme software, we must explain one of the mathematical techniques it uses extensively, namely the Seidel iterative solution of the input-output equations. In actual input-output computations, the Leontief inverse is seldom used, for the equations $q = Aq + f$ or $p = pA + v$ can be solved directly from the A matrix in about the same time required to multiply $(I - A)^{-1}$ by f or v. Thus, the effort of calculating $(I - A)^{-1}$ would be pointless. Moreover, for large matrices, many cells of A are zero. This fact can be exploited to reduce the computer storage required for the matrix. But the Leontief inverse will have non-zeroes nearly everywhere, so there is no way to reduce the space required for it. Further, changes to A are easily recorded and applied, but a change of one element in A can easily change all the elements in the inverse. Thus, from the point of view of solving the equations, nothing is gained and a good deal lost by computing the inverse.

How to solve the equations without the use of the inverse is the subject of this section. We will explain two methods of successive approximation, for it is worth knowing that both work even though we mainly use the second. The first, the simple iterative method, takes as a first approximation of q, $q_0 = f$. Then, given the nth approximation, q_n, the next approximation is

$$q^{n+1} = Aq^n + f \tag{15.2.1}$$

If the process converges so that one q is indistinguishable from the previous one, then the vector to which it has converged is clearly the solution of the equation. In economic terms, we first set the output equal to the final demands. Then we increase it to allow for the intermediate goods needed by the first approximation and then increase it again for the intermediate goods needed for the second approximation, and so on.

It is clear from equation (15.2.1) that if the matrix A is non-negative and f is non-negative, then no element of q ever becomes negative in the course of the iterations. Thus, the conditions on A that insure the convergence also insure that a non-negative f leads to a non-negative q. Thus, our inquiry, initially motivated by considerations of practical computation, also provides an answer to the theoretical question of whether an economy could exist with a given f and A, for the economic interpretation of Aq is dependent on all elements of q being non-negative.

The second method, the Seidel process, takes the same first approximation, and then, to get the second approximation, solves first the first equation for q_1, given all the other elements of q. Then, using this new value of q_1 and the old values of q_3, q_4, etc., solve the second equation for q_2, and so on. If the A matrix is triangular, that is, if all the entries above the main diagonal are zero, this method gives the right answer with one iteration. If it is not triangular, the whole process is repeated until little or no change occurs with each new iteration. While no actual input-output matrix is ever exactly triangular, the sectors can often be taken in an order which makes the matrix almost triangular, and this near triangularity speeds the convergence process.

Instead of starting this process with the final demands, it is also possible to start with any guess of q. In dynamic models, a good guess, namely the previous year's q is available. With a good starting point, four or five iterations of the Seidel process is usually sufficient to produce adequately accurate solutions. If twenty percent of the elements of A are non-zero -- a fairly typical situation -- we can make five iterations of the Seidel process in the same time which would be required to multiply f by the inverse if we had it.

If A is not an input-output matrix but just any old matrix you happen to meet on the street, there is

not much chance that either of these methods will converge and give a solution. What then makes us so sure that they will converge for an input-output matrix? To discuss *convergence*, we need to be able to say how far apart two vectors are. The concept of the *norm of a vector* gives us that ability. We even need to be able to say how far a given vector is from the solution when we do not know what the solution is. The concept of the *norm of a matrix* enables us to turn that trick. We will now explain these two concepts.

We can say how far apart two vectors are if we can say how "long" a vector x is, that is, how long the line is which connects x with the origin or zero point. For if $\|x\|$ represents the length of any vector, then the length of the difference of two vectors a and b, $\|a-b\|$, serves as a measure of how far apart they are. How shall we measure the length of a vector? In two dimensions, the usual length of the vector (x_1, x_2) is $\sqrt{x_1^2+x_2^2}$. This concept of length readily generalizes to vectors of any dimension by the definition $\|x\|=\sqrt{x'x}$. This formula, called the Euclidean length (or norm), gives one possible way of measuring length.

Why, however, do we bother to take the square root in the Euclidean norm? Because we certainly want *any* way of calculating the length of x to be such that multiplying each element of x by a scalar, λ, multiplies the length of x by the absolute value of λ:

(a) $\|\lambda x\|=|\lambda|\|x\|$

Other properties which any definition of length should have are:

(b) $\|0\|=0$ and $\|x\|>0 \; if \; x\neq 0$

and

(c) $\|x+y\|\leq\|x\|+\|y\|$

Property (c) expresses the requirement that the shortest distance between any two points must be a straight line. Let us denote the points by x and $-y$. Then we must have

$$\|x-(-y)\|\leq\|x\|+\|-y\|$$

since $\|x\|$ is the distance from x to 0 (the origin of the vector space) and $\|-y\|$ is the distance for 0 to $-y$, while $\|x-(-y)\|$ is the distance directly from x to $-y$. By applying property (a) to the second term on the right, this requirement may be written more simply as (c) above.

Any way of assigning a number, $\|x\|$, to each vector, x, of the vector space in such a way that (a), (b), and (c) are satisfied is called a *norm* of the space, and $\|x\|$ is read "the norm of x". It is quite remarkable that we can often prove the convergence of a process in terms of a norm without knowing exactly which norm we are using. Besides the Euclidean norm, there are two more important examples of norms:

the l-norm: $\|x\|=\sum_{i=1}^{n}|x_i|$

the m-norm: $\|x\|=max_i|x_i|$

You may easily verify that each of these norms has the required three properties, though the values they give as the norm of a given vector may be quite different. For example, the vector (1, -3, 2) has a Euclidean norm of 3.74, while its l-norm is 6 and its m-norm is 3. (The l in l-norm refers to Henri Lebesgue, a French mathematician of the early years of the twentieth century.)

Exercise 9: Draw the unit circle for each of these three norms. (The unit circle is the locus of points with norm 1.)

With each of these three norms, if x^k , for $k = 0, 1, 2$, etc., is a sequence of vectors and x* is a vector such that

$$\lim_{k\to\infty} \|x^k - x^*\| = 0$$

then

$$\lim_{k\to\infty} x^k = x^* \quad .$$

That is, convergence of a sequence of vectors in norm implies element-by-element convergence. This property is easily seen for the examples of the three norms and is a characteristic of finite dimensional vector spaces.

What we now want to show is that if q^* is a solution of the input-output equations, so that if

$$q^* = Aq^* + f \tag{15.2.2}$$

then the sequence $q^0, q^1, q^2, \ldots$ defined by

$$q^{k+1} = Aq^k + f \tag{15.2.3}$$

converges in norm to q^*. Subtracting the first equation, (15.2.2), from the second, (15.2.3), gives

$$q^{k+1} - q^* = A(q^k - q^*), k = 1,2,3 \ldots \tag{15.2.4}$$

If we have computed to iteration m, then setting $k = m$ in this equation gives

$$q^{m+1} - q^* = A(q^m - q^*)$$

But setting $k = m+1$ in (15.2.4) gives

$$q^{m+2} - q^* = A(q^{m+1} - q^*)$$

Together the last two equations imply

$$q^{m+2} - q^* = A(q^{m+1} - q^*) = A^2(q^m - q^*)$$

For any positive integer, p, similar reasoning applied p times gives

$$q^{m+p} - q^* = A^p(q^m - q^*) \tag{15.2.5}$$

We would like to be able to show that the norm of the vector on the left of (15.2.5) goes to zero as p goes to infinity. T o do so, we need to extend the concept of norm to matrices. We introduce that extension by a question:

Is there a number, call it $\|A\|$, such that

$$\|Ax\| \le \|A\| \ \|x\| \tag{15.2.6}$$

for all x?

There are indeed such numbers, and we call the least of them (for any norm of the vectors) the *norm*

of A. Intuitively speaking, the norm of the matrix A is the greatest "stretch" which multiplication by A performs on any vector. For the l-norm and m-norms of the vectors, the corresponding norms of a matrix are easily computed, as we shall see in a moment. Note that the norms of matrices also have the three same basic properties of the norms of vectors:

a) $\|A\|=0$ if and only if $A=0$.

b) $\|\lambda A\|=|\lambda|\|A\|$

c) $\|A+B\|\le\|A\|+\|B\|$

plus a fourth, which can be easily verified from the definition

d) $\|AB\|\le\|A\|\|B\|$

For the l-norm and m-norms of the vectors, the corresponding norms of a matrix are easilty computed, as we shall see in a moment. First, however, note that we can apply this inequality repeatedly to equation (15.2.5). After applying it p times, we have

$$\|q^{m+p}-q^*\|=\|A\|^p\|q^m-q^*\|$$

If we can show that $\|A\|<1$ for *some* norm, then for that norm

$$\|A\|^p\to 0 \; as \; p\to\infty$$

and therefore $q^k\to q^* \; as \; k\to\infty$ and the iterative calculations converge to the solution.

The norm of the n-by-n matrix A induced by the m-norm of vectors, and therefore called the m-norm of the matrix, is the maximum row sum, namely:

$$\|A\|_m=max_i\sum_{j=1}^{n}|a_{ij}|$$

while the norm of A induced by the l-norm of vectors, and therefore called the l-norm of the matrix, is the maximum column sum, namely,

$$\|A\|_l=max_j\sum_{i=1}^{n}|a_{ij}|$$

We shall prove the formula for the l-norm, and leave that for the m-norm as an exercise. (The Euclidean norm of A is more complicated and not of immediate concern to us. It is the largest characteristic root of $A'A$.) For the l-norm proof, let

$$\alpha=max_j\sum_{i=1}^{n}|a_{ij}|$$

Then $\|A\|_l\le\alpha$ because, for any x,

$$\|Ax\|_l=\sum_{i=1}^{n}|\sum_{j=1}^{n}a_{ij}x_j|\le\sum_i\sum_j|a_{ij}||x_j|=\sum_j|x_j|\sum_i|a_{ij}|\le\sum_j|x_j|\alpha=\alpha\|x\|_l$$

On the other hand, let k be the number of the column with the largest sum of absolute values, so that

$$\alpha=\sum_{i=1}^{n}|a_{ik}|$$

and then choose a vector, x, with $x_k=1$ and $x_j=0$ for all $j\ne k$. Then $\|x\|_l=1$ and

$$\|Ax\|_l = \sum_i |\sum_j a_{ij} x_j| = \sum_i |a_{ik}| = \alpha = \alpha \|x\|_l$$

Therefore, $\|A\|_l \geq \alpha$. But we have already shown the opposite inequality, so the only possibility is that $\|A\|_l = \alpha$.

If an input-output A matrix comes from an observed economy with a positive value-added in every industry, then the column sums of every column are less than 1 and therefore the l-norm of the matrix is less than 1. Thus, returning to the iterative solution of the input-output equations, we see that it will indeed converge if such is the source of A. Furthermore, in that case, $(I-A)^{-1}$ will be non-negative, because if we start from an f vector which is all zero except for a 1 in some position, the resulting solution will never have any opportunity to acquire any negative elements in the course of the iterative process. But the columns of $(I-A)^{-1}$ are precisely the solutions of such equations, so the whole matrix is non-negative.

The norm of the A matrix not only allows us to be sure that the iterative process converges, it also allows us to set an upper bound on how far we are from the solution at any stage. If, as before, q^k indicates approximation k, then

$$q^{k+p} - q^k = q^{k+1} - q^k + q^{k+2} - q^{k+1} + \ldots + q^{k+p} - q^{k+p-1} \qquad (15.2.7)$$

But since

$$q^{m+1} = Aq^m + f \quad \text{and} \quad q^m = Aq^{m-1} + f$$

for any positive integer m, subtraction of the second equation from the first gives

$$q^{m+1} - q^m = A(q^m - q^{m-1})$$

Repeatedly applying this equation gives

$$\begin{aligned} q^{k+1} - q^k &= A(q^k - q^{k-1}) \\ q^{k+2} - q^{k+1} &= A^2(q^k - q^{k-1}) \\ &\cdots \\ q^{k+p} - q^{k+p-1} &= A^p(q^k - q^{k-1}) \end{aligned}$$

and substitution in the above equation (15.2.7) gives

$$q^{k+p} - q^k = (A + A^2 + A^3 + \ldots + A^p)(q^k - q^{k-1})$$

Taking the norms of both sides and applying properties (c) and (d) of the norms of matrices gives

$$\begin{aligned} \|q^{k+p} - q^k\| &\leq \|A + A^2 + A^3 + \ldots + A^p\| \ \|q^k - q^{k-1}\| \\ &\leq \|A\| + \|A\|^2 + \|A\|^3 + \ldots + \|A\|^p \ \|q^k - q^{k-1}\| \end{aligned}$$

Now as p $\to \infty$, $q^{k+p} \to q^*$ and the sum of the geometric progression on the right goes to $\|A\|/(1-\|A\|)$ because $\|A\| < 1$. Thus, when we have reached iteration k, we know that the distance to the true solution is less than $\|q^k - q^{k-1}\| \|A\|/(1-\|A\|)$. In other words, when the differences of the successive approximations get small, we can be sure that we are close to the true solution.

Now suppose for a moment that A is a matrix in physical units -- with coefficients in units like kilowatt hours per pound -- so that column sums are meaningless and the l-norm perhaps much greater than 1. Further let w be an all-positive vector of the hours of labor -- the only primary input

-- required per physical unit of output in each industry. Can an economy exist with this technology? In other words, if the vector f of final demands is all positive, will the vector of outputs, q, such that $q = Aq + f$ also be all positive? (Mathematically, it is quite possible for some element of q to be negative, but it is economic nonsense to run an industry at a negative level. Coal can be converted into electricity, but all the electricity in the world can't make a ton of coal.)

The answer to these questions lies in the solution of $p = pA + w$ (where p is a row vector). If p is all positive, then it can be thought of as a vector of prices (with an hour of work as the numeraire) at which each process has a positive value added. If we now change the units of measurement of output of each product to one "hour's worth," the coefficient matrix, say A^*, in these new units corresponding to A in the old units will have columns whose sums are each less than 1. Thus, in these units, the iterative procedure will converge. But the iterative procedure in the original units (with A) would give successive approximations which differ from those with A^* only in their units. Hence the process would converge in the original units as well and will be non-negative. Since the Leontief inverse is non-negative, any vector of non-negative final demands can be met by non-negative levels of output of all the industries.

15.3. The Seidel Method and Triangulation

As mentioned at the outset of the previous section, there is a variation of the iterative method, known as the Seidel method, which converges even faster. In it, one starts with f as the initial guess of the solution just as in the simple iterative method, but then solves the first equation for the first variable and puts this value into the guess, then solves the second equation for the second variable and puts that value into the guess, and so on. Formally,

$$q_i^{(k+1)} = \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} + \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} + f_i / (1 - a_{ii})$$

In input-output work, the f vector is generally non-negative as are the elements of the A matrix. Hence, in the simple iterative method, the approximate solutions form a monotonically increasing sequence of vectors. The Seidel approximate solutions are also monotonically increasing but are always larger than the corresponding simple iterative solution. Hence, they also converge to the solution and do so faster than does the simple iterative method.

If all the non-zero elements of A are on or below the main diagonal, A is said to be triangular. If A is triangular, one pass of the Seidel process is sufficient to reach the exact solution. If A is merely almost triangular, a few iterations will suffice for a good solution. It general, input-output matrices arrive from the statistical offices more or less triangulated in exactly the wrong way. They start with Agriculture first, later Textiles, then Apparel. The right order for a fast Seidel solution is the reverse, Apparel, Textiles, Agriculture. It is not, however, necessary to physically re-arrange the rows and columns. All that is necessary is to take the rows in the Seidel operation in the order that would make the matrix nearly triangular.

For large matrices, however, it may be convenient to have a mechanical way to generate an approximately triangular order. A simple but effective is to pick as the first industry the one which has the smallest ratio of intermediate to final demand in its row. Then move into final demand all the inputs into this industry and again pick from the remaining sectors the one with the lowest ratio of intermediate to final in its row. Continue until all industries have been selected.

Solving input-output equations by the Seidel method is not only generally much faster than inverting the $I-A$ matrix by Gauss-Jordan reduction, it may even be faster than multiplying $(I-A)^{-1}$ by *f* when $(I-A)^{-1}$ is already known. How can that be? It is common for the A matrix to be quite sparse. A 300-by-300 matrix may have some 9,000 non-zero elements, not 90,000. It can be stored in a "packed" form in which only non-zero elements are stored, and the Seidel algorithm can be written to use this packed form, so that only as many multiplications and additions are required per iteration as there are non-zero elements. Thus, if the Seidel process requires less than ten iterations in our example, it will require less than 90,000 multiplications and additions. The Leontief inverse, however, will generally have 90,000 non-zeroes and thus multiplying it by *f* involves exactly 90,000 multiplications and additions. To economize on both space and solution time, large, sparse matrices are thus best stored in a packed form; and equations involving them should be solved by the Seidel process without ever inverting the matrix.

Exercises

10. Using C, C++, Fortran, Basic or any programming language you know, write a program to compute the triangular order of a matrix. Apply it to the flow matrix used as an example in this chapter. Write the results as a vector of integers, the first being the number of the equation to be taken first; the second, that of the equation to be taken second, etc.

11. Write a program to use the Seidel method to solve input-output equations, taking the equations in the order specified by the vector produced in exercise 7. Apply the program to solve exercise 1 earlier in this chapter. (*Bump* has a Seidel method. Try to create yours without looking at it.)

CHAPTER 16. BUILDING MULTISECTORAL MODELS WITH INTERDYME

16.1. Introduction to Interdyme

In section 15.1, we became acquainted with the VAM file and saw that *G7* could do a number of calculations with the matrices and vectors in these files. By the end, however, we came up against the limits of building models in *G7* by itself. To integrate regression equations and input-output we proceed to Interdyme. Interdyme makes use of the Seidel process for solving the input-output equations, so we had to explain that before getting into Interdyme itself. Starting in this section, we will build several successive versions of the *Tiny* model in Interdyme. We will being in this section with the version called "Model 1".

Interdyme is a collection of C++ programs which make it easy to construct interindustry dynamic models involving regression equations, input-output computations with matrix algebra, and lag relationships that provide dynamics. In this section, we will introduce Interdyme by building a simple Interdyme model of *Tiny* with just one regression equation and rudimentary institutional accounts.

We are going to assume that the historical period for which we have data extends up through 2003 and that the forecast period will be 2004 – 2010. At this point, the HIST.VAM has been worked with considerably and may have things in it which would prove confusing, so we will rebuild it from scratch in *G7* by doing

```
add tiny.pre
```

where TINY.PRE is the file shown in the box on the next page. This file will fill in all the matrices and vectors for the historical period, but in the forecast period only the coefficients are filled in, not the input-output flows. In particular, output is not calculated in the forecast period. Thus, if – after Interdyme has been run – we find data for industry output in the forecast period, we can be sure that it was calculated by Interdyme.

Regression Equations and Accounting Identities

We can conveniently begin with the most familiar part, the regression equation and the accounting identities. The regression equation is estimated in *G7* by the following command

```
add invtot.reg
```

where invtot.reg is the following file

```
catch invtot.cat                # Open up the catch file
save invtot.sav                 # Open up the save file
ti TINY investment equation     # Title for regression or graph
dfreq 1                         # Set default frequency to 1 (annual)
f ub20 = @cum(ub20,1.,.20)      # Form a "unit bucket" for capital stocks
f capstk = @cum(invcum, invtot[1], .20)/ub20    # Calculate capital stock
f delGDP = gdp - gdp[1]         # First difference of GDP
con 2000 1.03 = a1              # Soft constraint for regression
lim 1982 2003                   # limit of the regression
r invtot = ! capstk, delGDP[1], delGDP[2]   # regression with no constant term
save off                        # close the save file
```

The TINY.PRE File

```
# tiny.pre - File to create VAM file for Interdyme TINY
vamcreate vam.cfg hist
vam hist b
dvam b
# Bring in the intermediate flow matrix
add flows.dat
# Bring in the final demand vectors
add fd.dat
# Bring in the value added vectors
add va.dat
fdates 2000 2000
# Add up the intermediate rows
getsum FM r out
# Add on the final demand vectors to get total output
vc out = out+pce+gov+inv+ex+im
# Copy intermediate flows to AM and convert to coefficients
mcopy b.AM b.FM
coef AM out
# The following are element-by-element vector divisions
vc depc = dep/out; vc labc = lab/out; vc capc = cap/out; vc indc = ind/out
# Compute share vectors of final demands
vc pcec = pce/pcetot{2000}; vc invc = inv/invtot{2000}
vc govc = gov/govtot{2000}; vc exc  = ex/extot{2000}
vc imc = im/imtot{2000}
fdates 1995 2010
# Copy the 2000 AM matrix into 1995 - 2010
dfreq 1
f one = 1.0
index 2000 one AM;
# Create a time trend
f time = @cum(time,one,0)
# Move the five final demand columns by their totals in the historical
#   years, 1995-2003
fdates 1995 2003
index 2000 pcetot pce; index 2000 invtot inv;  index 2000 govtot ov
index 2000 extot ex; index 2000 imtot im
# Keep value added and final demand coefficients constant
fdates 1995 2010
f one = 1.
index 2000 one depc; index 2000 one labc
index 2000 one capc; index 2000 one indc
index 2000 one pcec; index 2000 one invc; index 2000 one govc;
index 2000 one exc;  index 2000 one imc
# The remaining calculations are done for the historical period
#   1995 - 2003 only
# Take the Leontief inverse of the A matrix
fdates 1995 2003
mcopy b.LINV b.AM
linv LINV
# Add up the final demands
vc fd = pce+gov+inv+ex+im
# Compute total outputs
vc out = LINV*fd
# Computer value-added flows
vc dep=depc*out; vc lab=labc*out; vc cap=capc*out; vc ind=indc*out
store
close b
# We should now have outputs and final demands in the historical period, but only coefficients in the forecast period.
```

```
gr *                          # Graph of the regression fit
catch off                     # Close the catch file
```

with the following results (from the catch file)

```
:                              TINY investment equation
 SEE   =      13.51 RSQ   = 0.8360 RHO =   0.60 Obser  =   22 from 1982.000
 SEE+1 =      11.45 RBSQ  = 0.8187 DW  =   0.79 DoFree =   19 to   2003.000
 MAPE  =       7.24
   Variable name           Reg-Coef  Mexval  Elas   NorRes     Mean   Beta
 0 invtot                - - - - - - - - - - - - - - - - - -    161.06 - - -
 1 capstk                   0.99562   402.8   0.89    1.52    144.50
 2 delGDP[1]                0.35900    17.0   0.11    1.00     47.67  0.231
 3 delGDP[2]                0.03007     0.1   0.01    1.00     47.06  0.020
```

The national accounts are given by the file ACCOUNT.SAV as follows:

```
# The Accountant for Tiny
# Personal interest and dividends
fex pintdivrat = pintdiv/capinc
f pintdiv = pintdivrat*capinc
# Personal income
f pi = labinc + pintdiv + pgovtran
# Personal taxes
fex ptaxrat = ptax/pi
f ptax = ptaxrat*pi
# Personal disposable income
f pdisinc = pi - ptax
# Personal saving
fex psavrat = psav/pdisinc
f psav = psavrat*pdisinc
f pcetot = pdisinc - psav
# Government income
f ginc = indtax + ptax
# Government saving
f gsav = ginc - govtot - pgovtran
# Business saving
f bsav = capinc - pintdiv - invtot
# RoW saving
f RoWsav = imtot – extot
```

All of this file should be familiar from the Master file of macromodels and the previous discussion of the Institutional accounts in this chapter.

IdBuild, the Interdyme Version of the Build Program

Both of these .SAV files will be passed through the *IdBuild* program, the Interdyme version of the *Build* program. It is much easier to run *IdBuild* than to explain what all it does. Basically, *IdBuild* converts the *G7*-estimated parts of the model into C++ code. Clicking Model | IdBuild not only runs *IdBuild* but also compiles the C++ code it has written and combines it with the other C++ modules of the Interdyme system to make an executable program. Thus, running *IdBuild* just requires two mouse clicks. Recounting what it does will take us a little longer, but some understanding of the results is necessary to play your creative role in writing MODEL.CPP that pulls together the pieces

of the Interdyme system.

First, we must note that *IdBuild* is not so omniscient as *Build* and needs our help on one point. *Build* knows about all the variables and equations in the macromodel it is building. Therefore, it can arrange for getting all of them into the model, can figure out which ones are endogenous, and can write a file for making quick projections of all those which remain exogenous. Because part of an Interdyme model may be written directly in C++ without going through *IdBuild*, *IdBuild* does not necessarily know about all the variables in the model and does not know about the possible definition of some variables in the C++ code. A file called PSEUDO.SAV is used to inform *IdBuild* of the variables which are used in the C++ code or otherwise needed in the data bank produced by *IdBuild* but not used in any of the .SAV files processed by *IdBuild*. In *Tiny*, there is only one variable of this sort, *timet*, which is used in the regressions for projecting exogenous variables. The second function of the PSEUDO.SAV file is to tell *IdBuild* which variables will be defined in the C++ code and therefore do not need regressions in EXOGALL.REG to make quick, mechanical projections. In *Tiny*, there are five such variables, as we shall see below, called *gdp*, *deprec*, *labinc*, *capinc*, and *indtax*. Here is the PSEUDO.SAV file for *Tiny*:

```
# PSEUDO.SAV file for TINY
# Time
f timet = timet
# GDP Gross domestic product
f gdp = gdp
# Depreciation income
f deprec = deprec
# Capital income
f capinc = capinc
# Labor income
f labinc = labinc
# Indirect taxes
f indtax = indtax
```

With these files in place, we are ready to look at the MASTER file, which is just the following:

```
#  Master File for TINY
iadd invtot.sav
iadd account.sav
iadd pseudo.sav
end
```

The only commands in the Master file for *IdBuild* are comments, “iadd” commands to bring in the various .SAV files, and the end command to signal the end of the commands.

IdBuild requires a configuration file called BUILD.CFG. In it, I recommend that you make the name of the workspace bank HIST, which is the way the file is supplied.

Once these files are ready, *IdBuild* can then be run. I would suggest that, for the moment, you do so from within *G7* by the command

```
dos c:\pdg\idbuild master
```

Later, when you have written the MODEL.CPP file, you can run *IdBuild* and compile and link MODEL.CPP and other parts of the Interdyme program by clicking Model | IdBuild. We will get to

that point soon. Right now we should have a quick look at the output of *IdBuild*. It is reminiscent of that of *Build*. The EXOGALL.REG file, for example, begins as:

```
bank hist b
mode f
tdates <StartType> <EndType>
limits <StartReg> <EndReg> <EndForecast>
ti govtot
r b.govtot = timet
gr *
f govtot = depvar
save govtot.xog
sty govtot
save off
```

As with a macromodel, areas marked with the <...> in the third and fourth lines must be replaced by dates and the file then passed through *G7* to generate for each variable a .XOG file with exogenous forecast for each of the exogenous variables. The file RUN.XOG will contain an "add" command for each of these files. For *Tiny*, RUN.XOG is

```
dos copy hist.* base.*
wsb base
add pintdivrat.xog
add pgovtran.xog
add ptaxrat.xog
add psavrat.xot
add govtot.xog
add extot.xog
add imtot.xog
add pgovtran.xog
add pintdivrat.xog
add ptaxrat.xog
add psavrat.xog
wsb ws
[possibly also lines, not needed in TINY, to project exogenous vectors in
the vam file, like this:
vam base b
dvam b
vmatdat …
]
```

Note the effect of the first two and the last commands when executed in *G7*. As noted above, HIST is the recommended name (specified in BUILD.CFG) for the workspace bank generated by *IdBuild*. The first command above copies this G bank and VAM file to a G bank and VAM file with the name BASE. The second line makes this bank the workspace bank of *G7*. The subsequent lines add the files with the projections of all the exogenous variables. These projections go into the G workspace bank, namely, into BASE. The "wsb ws" command makes WS the workspace bank of *G7* and thereby frees the BASE bank. As in the case of macromodels, you should check the mechanical exogenous projections, shape them to your preferences, and put revised versions into files with the extension .XG, and make a revised RUN.XOG, calling it something like BASE.XG. For the moment, we will let well enough alone and just use RUN.XOG with the mechanically generated projections. This would also be the appropriate place to put into BASE.VAM the forecasts of any

exogenous vectors or matrices not already projected by running TINY.PRE. (In *Tiny*, there are no such vectors or matrices.) Remember, however, that if you make any changes to RUN.XOG you should change the name to RUN.XG or something more descriptive. Otherwise, the next time *IdBuild* is run, your carefully prepared RUN.XOG will be overwritten.

Like *Build*, *IdBuild* also writes RUN.GR, a command file for *G7* which will graph all timeseries variables handled through *IdBuild*.

Running *IdBuild* generates a number of other files to facilitate the writing of the C++ program to run the model. One of these is TSERIES.INC as follows:

```
GLOBAL Tseries ub20, invtot, invcum, capstk, gdp, delGDP, pintdivrat,
capinc, pintdiv, labinc, pgovtran, pi, ptaxrat, ptax, pdisinc, psavrat,
psav, pcetot, indtax, ginc, govtot, gsav, deprec, bsav,
imtot, extot, RoWsav, timet;
```

This is just a listing of all the macro variables in the model in a format suitable for use in the model.

Like *Build*, *IdBuild* also writes a file of C++ code called HEART.CPP. It is shown in the box on the next page, in somewhat abbreviated form, for *Tiny*. Wherever four dots occur, thus .... , lines have been omitted which just repeat for the other variables listed above in TSERIES.INC the same function above and below the four dots for *invtot* and *ginc*. The Master file had the two following lines:

```
iadd invtot.sav
iadd account.sav
```

Each of these resulted in a corresponding function of C++ code in the HEART.CPP file, namely *invtotf()* and *accountf()*, respectively. These are functions which can be called from MODEL.CPP, the user-written part of Interdyme system. The function name is created by adding an 'f' to the end of the name of the file. A glance at *accountf()* shows that it is just an adaptation for C++ of the G commands in ACCOUNT.SAV. The function *invtotf()* is the same for INVTOT.SAV with the extra feature that the numerical values of the regression coefficients have been stripped off and put in a separate file, HEART.DAT, which will automatically be read to supply values to the *coef* variable. The regression coefficients are treated this way so that they can potentially be varied when optimizing the fit of the model, just as was done with macromodels. *IdBuild* recognizes the special function of the line

```
iadd pseudo.sav
```

in the MASTER file and does not generate a corresponding function in HEART.CPP.

Also in the HEART.CPP file are the C++ functions *tserin()* and *uptseries()*. The first of these reads in the historical values and exogenous projections of the time series variables; it is executed at the beginning of any run of the model. The second, *uptseries()*, is called at the beginning of the calculation of the forecast for any year. It looks at the starting value of each of the time series variables and if it is -.000001, the value *G7* uses to indicate a *missing value*, then it replaces that missing value with the value of the series in the previous year. For example, in *Tiny*, no exogenous projection is made for Personal income, since it is endogenous. Thus, its starting value in 2004 would be -.000001. If the model starts with this very bad guess of Personal income, it eventually

converges to the correct value, but it will converge much faster if it starts with a good guess, and the previous year's value is usually a pretty good guess. Note that exogenously projected values will not be affected, because they will not be -.000001.

The HEART.CPP File for *Tiny*

```
#include "dymesys.h"
#include "heart.h"
extern short t;
extern float **coef;
FILE *fmatrix;
float depend;
#include "tseries.inc"
/* end of standard prolog */
void invtotf()
{
    /*  TINY investment equation */
    ub20[t]=cum(ub20,1.,.20);
    capstk[t]=cum(invcum, invtot[t-1],.20)/ ub20[t];
    delGDP[t]= gdp[t]- gdp[t-1];
    /* invtot */ depend = coef[0][0]*capstk[t]+coef[0][1]*delGDP[t-1]+coef[0][2]*delGDP[t-2];
    invtot.modify(depend);
}
void accountf()
{
    pintdiv[t]= pintdivrat[t]* capinc[t];
    pi[t]= labinc[t]+ pintdiv[t]+ pgovtran[t];
    ptax[t]= ptaxrat[t]* pi[t];
    pdisinc[t]= pi[t]- ptax[t];
    psav[t]= psavrat[t]* pdisinc[t];
    pcetot[t]= pdisinc[t]- psav[t];
    ginc[t]= indtax[t]+ ptax[t];
    gsav[t]= ginc[t]- govtot[t]- pgovtran[t];
    bsav[t]= capinc[t]- pintdiv[t]- invtot[t];
    RoWsav[t]= imtot[t]- extot[t];
}

void tserin()
{
    timet.in("timet");
    invtot.in("invtot");
    ....
    ginc.in("ginc");
}
void uptseries()
{
    //Function to replace missing values or macro vrariable with lagged values.
    if(timet[t]< .0000009 && timet[t]> -.0000011) timet[t] = timet[t-1];
    if(invtot[t]< .0000009 && invtot[t]> -.0000011) invtot[t] = invtot[t-1];
    ....
    if(ginc[t]< .0000009 && ginc[t]> -.0000011) ginc[t] = ginc[t-1];
}
```

A final file created by *IdBuild* is CALLALL.CPP , a bit of C++ code which calls all of the functions written by *IdBuild* in the HEART.CPP file. It is used in conjunction with managing rho adjustments as will be explained below in conjunction with the MODEL.CPP file. Here is the important part of CALLALL.CPP for *Tiny*.

```
void callall()
{
invtotf();
accountf();
```

```
}
```

Writing MODEL.CPP for Tiny

So far, there has been a lot of similarity between building Interdyme models and building macromodels with *G7* and *Build*. Now we venture into new territory with the writing of the MODEL.CPP file. The box on the following page shows the part of this file that distinguishes *Tiny* from any other model built with Interdyme. I have more than once encountered the reaction, "C++ is hard; I don't have time to learn it, so I'll just stay with the models I can build in Excel." It is true that some of the advanced features of C++ can be rather arcane, but the objects such as Vector, Matrix and others used in Interdyme are already written, in separate files. The code that you will need to write is in no more complicated than the simplest level of Basic, Fortran, or C. But with the Interdyme infrastructure, you can easily write the code for the matrix and vector operations that are essential for working with input-output models but are tedious to program in those other languages.

The box shows the C++ code that defines *Tiny*. First of all, you need to know a few things about C++ grammar. Anything following a // on the same line is a comment, useful for humans but ignored by the computer. A comment extending over more than one line is begun with a /* and ended with a */. Every C++ statement ends with a semicolon. More than one statement can be put on one line or a single statement can be broken onto two or more lines between words. C and C++ are case sensitive: *x* is not the same as *X*. For any variable *x*, x++ means "add 1 to *x*." Before a variable name, like *Iteration* or *oldinvtot*, can be used, we must declare what kind of variable it is by statements like

```
int Iteration; // declare Iteration to be an integer
float oldinvtot; //declare oldinvtot to be a floating point, real number
```

A group of statements, indicated by enclosing the statements in curly braces like these { }, can be used anywhere a single statement could be used, notably in **for**, **do**, **while**, **if**, **else**, and **else if** constructions. The **for** loop that fills most of the box of *Tiny* code is a good illustration of this point. It looks something like this:

```
for (t = godate; t<= stopdate; t++) {
    ....
    }
```

where the .... represents many lines of code. The command means: start *t* off equal to *godate*, which is the beginning year of the run you are making, something like 1995, then do all the statements represented by the .... with that new value of *t*, then increment *t* by 1 – that's the t++ near the end of the line – and do all the statements represented by the .... with that value of *t*, and keep doing all this as long as *t* is less than or equal to *stopdate*, a number that other parts of Interdyme will have made equal to the last year of your run, something like 2010. Now the generic form of the for loop is written

for (*initialization, condition, increment*) *statement*

The Core of TINY's MODEL.CPP File

```
for (t = godate; t<= stopdate; t++) {
   if (MaxFlag == 'n') printf("%d ",t);
   // Load all vectors and matrices.
   load(t);
   uptseries();
   // Start of code particular to TINY:
   Iteration = 0;
   // The loop for convergence on pcetot and invtot.
   while(Iteration < 20){
       Iteration++;
       oldinvtot = invtot[t]; oldpcetot = pcetot[t];
       if(t>= MacEqStartDate)
          invtotf();
       inv = invtot[t]*invc;      pce = pcetot[t]*pcec;
       gov = govtot[t]*govc;      im  = imtot[t]*imc;
       ex  = extot[t]*exc;
       // Add up final demand vectors
       fd = pce + gov + inv + ex + im;
       // Solve input-output equations by Seidel method
       Seidel(AM, out, fd, triang, toler);
       // Compute value-added vectors
       // The Interdyme ebemul() function does element-by-element multiplication
       dep = ebemul(depc,out);  lab = ebemul(labc,out);
       cap = ebemul(capc,out);  ind = ebemul(indc,out);
       // The Accountant for Tiny
       gdp[t] = fd.sum();
       if(t>MacEqStartDate){
          deprec[t] = dep.sum(); labinc[t] = lab.sum();
          capinc[t] = cap.sum(); indtax[t] = ind.sum();
          accountf();
          }
       // Form the convergence test
       invdif = fabs(invtot[t] - oldinvtot);
       pcedif = fabs(pcetot[t]- oldpcetot);
       printf("Iter %2d pce = %7.1f pcedif = %6.2f invdif = %6.2f\n",Iteration,
          pcetot[t],pcedif,invdif);
       if(invdif < .5 && pcedif < .5) break;
       }
   // End of code particular to TINY
   // Here when both Investment and PCE have converged
   // Standard end of the spin() function:
   if(MaxFlag == 'y')
           shiftback(t);
   else{
       // Store the values of vectors and matrices for this period.
       store(t);
       printf("\n");
       }
   }
```

It starts off one or more variables in the *initialization*, checks that the *condition* is true and, if it is, executes the *statement*, then revises the variable that was initialized as prescribed by the *increment*, checks the condition, and executes the *statement*, and so on as long as the *condition* is true. When the *condition* is no longer true, control passes to the next statement below the **for** loop. How the *initialization*, *condition*, and *increment* work in the example is clear enough, but our particular point

here is that the *statement* executed is the compound statement composed of all the simple statements enclosed by the braces that open right after the closing parenthesis of the for statement. I should also mention that indentation in C and C++ is solely for the benefit of the human reader; it doesn't matter to the computer.

The construction

if (*condition*) *statement*
else if (*condition*) *statement*
else *statement*

works in an entirely similar way. Any number of **else if** lines may be used. In writing the conditions, any of the following operators may be used:

== is equal to
!= not equal
< is less than
> is greater than
>= is greater than or equal
<= is less than or equal
| | or
&& and
! not

Besides the **for** keyword, the keywords **while** and **do** can also be used to write loops. The general form of the **while** is

while(*condition*) *statement*

which executes the statement as long as the condition is true. There is an example in the *Tiny* code:

```
while(Iteration < 20){
```

For the **do** keyword, the syntax is

do *statement* **while** (*condition*);

The *statement* executes until the *condition* becomes false. Since the *condition* is tested after each pass through the loop, the loop will execute at least once. In practice, the **do** loop is seldom used.

Finally, we need to mention three statements for jumping about in the code. The most commonly used is **break**, which breaks out of the innermost loop where it is found. There will be an example in the *Tiny* code below. The **continue** statement shifts control to the end of the innermost loop, while

goto *label;*

sends control to the *label*. The *label* is a line with one word ending in a colon, like *top*: or *finish*: . In good C++ style, the **goto** is seldom used; but it is useful when breaking out of a deep nest of loops. (If you have ever used Fortran, please note that the function of the C++ **continue** statement is totally unlike that of the Fortran CONTINUE statement.)

That is about all you need to know about C++ in general to use Interdyme. We can now turn to the C++ code in the box. First of all, there are a few Interdyme-specific functions and variables you need to know about. In the second line of code, we meet the variable *MaxFlag*; it is a single letter, either 'y' or 'n' for "yes" or "no". It will be 'y' only if the user has specified that we are to do an optimization. We will assume that it is 'n'. In that case, the *printf()* function writes to the screen the year which is about to be computed. (For the full capabilities of *printf()* or the C++ alternative keyword out, you can consult the help file of your C++ compiler.)

The program then calls the Interdyme functions *load(t)* to load into the computer's random access memory (RAM) from the computer's hard disk the starting values for year *t* of all vectors and matrices in the VAM file on the hard disk. At the bottom of the for loop, the *store(t)* function stores their newly computed values back to the VAM file.

Besides the matrices and vectors handled by the VAM file, most Interdyme models will also have macro variables such as GDP, total labor force, total employment, unemployment, or an interest rate, which for any given period, have only one number – not a vector or matrix of numbers – as their value. Because such variables occur also in macroeconomic models, we refer to them as *macro variables*, even though they may be quite "small" or very specific variables, such as "children under 5 years of age". They are carried in a G bank which generally has the same root name as the corresponding VAM file. Similarly, we will refer to as *macro equations* all of the regression equations handled one-by-one, as in macro models.

The call to the function *uptseries()* at the beginning of MODEL.CPP checks all the macro variables, and where it finds a value missing in the current period, period *t,* it inserts the value of that macro variable from the previous period. As noted above, this replacement gets the iterative process of solution off to a good start. On the other hand, exogenous variables that have specified values are not affected by the call to *uptseries()*. The *uptseries()* function was written by *IdBuild* and is in the file HEART.CPP.

At this point, we begin the code that is particular to *Tiny*. Through the general Interdyme structure, the person running the model provides a value for the variable *MacEqStartDate*, the date at which the macro equations of the model begin to be computed. *Tiny* has only one macro equation, namely that for *invtot(t)*, total investment. Since we have known accounts up through 2003, we will set *MacEqStartDate* equal to 2003. (How we do so, we will see shortly.) Prior to this date, the Interdyme model for *Tiny* produces the same results as did the *G7*-only model. In 2003, only *invtot* and *gdp* may be different. Other macro variables are not affected because the national income accountant function, *accountf()*, is not called until the next year. In practice, we will normally set the rho-adjustment factor for all of the macro variable regression equations in this year (how that is done will be explained below). Those variables will not differ from historical values in this last year of historical macroeconomic data.

Once *t* has passed the *MacEqStartDate*, the final demands determine output (via the Seidel input-output calculations), but output determines value-added which in turn determines (via the institutional accounts) Personal income. This thenv determines total Personal consumption expenditure, which is one of the final demands. Just as in macro models we solve this circularity by iteration. That is, we start off with one set of final demand totals – *pcetot[t]*, *invtot[t]*, *govtot[t]*, *extot[t]*, *imtot[t]* – and compute along until will have calculated new values. We then compare the new values with the old, and if the differences exceed the tolerances we set, we go back to compute

with the new values of the variables, final demands, outputs, value added, and the variables just listed. We repeat the process until convergence is reached or the iteration counter exceeds 20, whereupon we go on to the next year. Since *govtot[t]*, *extot[t]* and *imtot[t]* are exogenous, they will not vary from iteration to iteration, so only the values of *pcetot[t]* and *invtot[t]* are checked for convergence. In fact, the form of the regression equation we estimated has no dependence of *invtot[t]* on variables in period *t,* so we do not for the present model need to check *invtot[t]*, but we leave the check because in principle there could be such a dependency. The lines of code that drive the looping process are:

```
Iteration = 0;
// The loop for convergence on pcetot and invtot.
while(Iteration < 20){
   Iteration++;
   oldinvtot = invtot[t]; oldpcetot = pcetot[t];

   /*****************************************************************
      The substance of the model, which follows here, has been cut out to
      emphasize the testing of convergence. This substance computes new
      values of invtot[t] and pcetot[t].
      *****************************************************************/

   // Form the convergence test
   invdif = fabs(invtot[t] - oldinvtot);
   pcedif = fabs(pcetot[t]- oldpcetot);
   printf("Iter %2d pce = %7.1f pcedif = %6.2f invdif = %6.2f\n",Iteration,
      pcetot[t],pcedif,invdif);
   if(invdif < .5 && pcedif < .5) break;
   }
```

The first line initializes an integer, *Iteration*, to 0. The line

```
while(Iteration < 20){
```

sets up the loop that iteratively solves the circularity just explained. The first line within the loop increments *Iteration* by one, so we see immediately that this loop will not be executed more than 20 times. We then store away the starting values of *invtot[t]* and *pcetot[t]*. Then we execute the substance of the model, which results in changing these values. When that work is complete, we use the standard C++ function *fabs()* to take the absolute value of the difference between the previous and the new values of these two variables. If both of those values are less than .5, we break out of the **while** loop. More complicated models may have more complicated convergence tests, but *Tiny's* example is pretty typical.

Once convergence has been reached, the main thing to be done is to call *store(t)*, which stores back to the computer's hard disk this period's values of the vectors and matrices.

With the iterative solution of the circularity understood, we turn to the the substance of the model. It is easily followed in the code in the box below. The first thing we see is the call to *invtotf()* when *t* is greater than or equal to the starting date for macro equations. The next three lines show multiplication of a vector (such as *invc*) by a scalar (such as *invtot[t]*). Then follows the addition of five vectors to form the vector *fd*. Then the function *Seidel()* is called to compute the output vector *out* implied by the final demand vector *fd*, and the input-output coefficient matrix *AM*. The the value-added vectors are computed via element-by-element multiplication of the coefficient vectors

with the output vector. Then *gdp* and the value-added component totals are computed as sums of vectors. Finally, the accountant is called to make up the national accounts.

Substance of MODEL.CPP for TINY

```
if(t>=MacEqStartDate)
   invtotf();
inv = invtot[t]*invc;  pce=pcetot[t]*pcec;
gov = govtot[t]*govc;  im = imtot[t]*imc;
ex = extot[t] * exc;
fd = pce + gov + inv + ex + im
Seidel(AM, out, fd, triang, toler);
dep = ebemul(depc,out);  lab = ebemul(labc, out);
cap = ebemul(capc, out); ind = ebemul(indc, out);
// The Accountant for Tiny
gdp[t] = fd.sum();
if(t>MacEqStartDate) {
   deprec[t] = dep.sum();  labinc[t] = lab.sum();
   capinc[t] = cap.sum();  indtax[t] = ind.sum();
   accountf();
   }
```

This code has several vector and matrix operations. The Interdyme code, nestled away in other modules (.CPP files) where you do not need to bother reading it, defines what a Vector and Matrix are and instructs the computer how to read them, add or subtract them, multiply a Matrix by a Vector, sum up the elements of a Vector, and do both inner-product and element-by-element multiplication of two Vectors. Let me emphasize: Matrix and Vector are not general C++ features; it is the other modules of Interdyme working in the background that give you these handy tools for writing the code for input-output models. In fact, though we have used here only vector addition, Interdyme has a rather complete library of matrix and vector routines.

The Interdyme function *Seidel(AM, out, fd, triang, toler)* solves by the Seidel method the equation

*out=AM*out + fd*

using the order of equations specified by the integer vector *triang* and with the convergence tolerance specified by *toler.* The value of integer vector, or *Ivector,* is given by the file TRIANG.IV. For *Tiny*, its value is

8 7 5 6 4 3 1 2

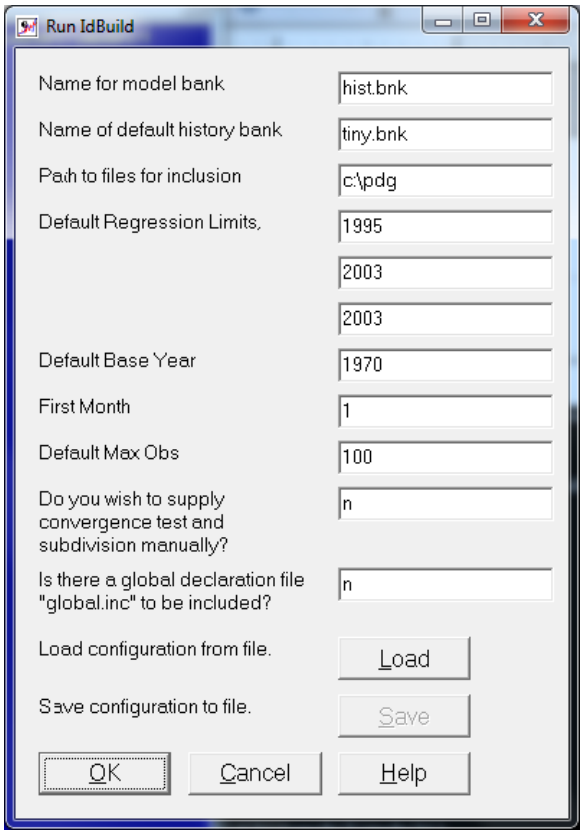
The Interdyme function *ebemul()* does an element-by-element multiplication of two Vectors and gives a Vector as its output. The floating point absolute value function, *fabs()*, as already mentioned, is a standard C++ function.

Running the Interdyme Model

With the code of MODEL.CPP ready to go, we can create the G bank with all the macrovariables of the model, write the C++ code for handling the regression equations and identities, compile this code and MODEL.CPP, link the compiled modules (whose file names end in *.obj*) into an executable program (*.exe*) and get started on forecasting the exogenous variables, all by clicking "Model" in the *G7* main menu and then "Run IdBuild and compile model".

These clicks bring up the window shown below. If there was a BUILD.CFG file in the directory where *G7* was started, then it will have been used to fill in the boxes in this form.

Run IdBuild Dialog



Otherwise, the user can fill them in, click OK, and the content will be written as the BUILD.CFG file. The first box, labeled "Name for model bank", contains the name of the G bank to be created having all the variables in the model, but no others. The second box, labeled "Name of default history bank", contains the name of a G bank from which to draw historical values of the variables, until a "bank" command switches to a different input bank. The first bank is created by the *IdBuild* program; the second must already exist prior to running *IdBuild*. Generally, the G bank created by *IdBuild* should have the same root name as the VAM file with the historical values of the vectors and matrices.

The next three fields labeled "Default regression limits", are not actually used. They are there to

keep the format of BUILD.CFG the same as that of G.CFG files; any dates will work fine. Then come the base year of the bank to be created, the first month of that year which is covered (which is always 1 for annual models), and the maximum number of observations (years) to be accommodated in the bank. The next two fields allow for advanced features we will not be using and should for our purposes just contain the letter *n* for *no*. When the form has been filled in as desired – and it will usually automatically be filled in correctly – click OK.

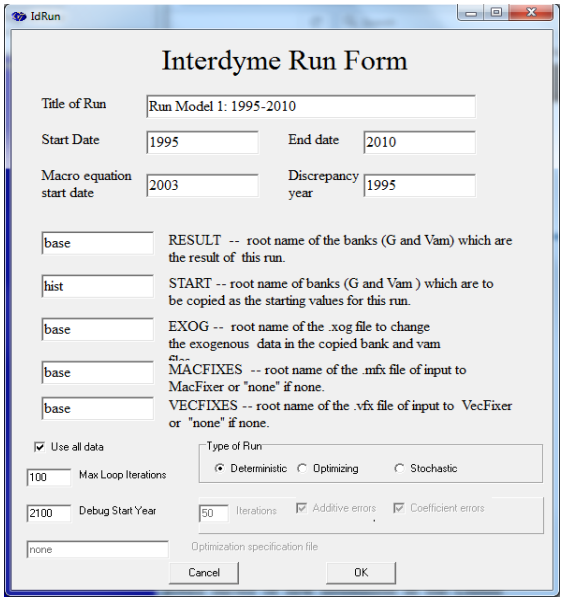
This one click runs *IdBuild*, which writes HEART.CPP. It also compiles MODEL.CPP, HEART.CPP and any other modules that need to be compiled, links them together and produces DYME.EXE, the executable file for running the model.

As described in the earlier section, running *IdBuild* also writes EXOGALL.REG and RUN.XOG. It also creates the HIST bank of macro variables in the model. You can use EXOGALL.REG to make projections of the exogenous variables, exactly as with macro models. It will put the predicted values of each exogenous variable into its own .XOG file, such as this one, GOVTOT.XOG

```
update govtot
    2003    348.1439   353.9075   360.3546   367.3013   374.6133
    2008    382.1923   389.9666   397.8835
```

As with macromodels, you can edit these .XOG files to change the forecasts. In doing so, you can make use of any function in G7.[4] When the file is edited, change the extenstion of the filename from .XOG to .XG so that your carefully edited file is not overwritten the next time *IdBuild* is run.

For the present, we are going to content use the RUN.XOG as automatically written (and shown above) and also with all the .XOG files. But we need a copy of RUN.XOG named RUN.XG. We can do that by simply opening RUN.XOG in the editor and saving it as RUN.XG. For our first example, we will also copy RUN.XG to BASE.XG, and this file will be used in the discussion below.



You can now run the Interdyme model by clicking

Model | Run Dyme Model

The form shown on the right then appears.

The "Title of Run" can be anything that fits in the space provided. It will show up on tables made with the *Compare* program.

4 But the convenient device of putting a 0 for values to be linearly interpolated does not work because in *G7* it is sometimes important to give a true 0 to a variable. Instead, in the "update" command, one leaves out the value to be linearly interpolated and uses *G7*'s @lint() function. For example, if we wanted to keep the above values of *govtot* for 2003 – 2007 but set the 2010 value to 420, and interpolate 2008 and 2009 linearly between 2007 and 2010, we would put into GOVTOT.XG the following:

```
update govtot
2003 348.1439  353.9075  360.3546  367.3013
2007 374.6133
 2010 420
f govtot = @lint(govtot)
```

The “Start Date” field should be a four-digit year, for example, 2008. It is the year in which the calculations begin. The “End date” field is similar; it is the last year for which calculations will be made.

The “Macro equation start date” is the first year in which the macro equations will be calculated. In forecasts, it will generally be the last year for which there is data on values of the macro variables. This data is then used to set the initial value of the rho adjustment. For historical or counter-historical simulations, the macro equation start date is some time before the last available data.

The “Discrepancy year” is used in some models as the year for calculating a discrepancy vector subsequently used in the input-output calculations. If your model does not use this device, just fill in any year.

The next five fields all use the concept of the *root name* of a file. It is the part of the filename following the directory name, but before the dot in the name. Thus, the root name of the file whose full name is C:\MALAYSIA\MODEL\BASE.BNK is just BASE. The part of the file name after the dot is called the *extension*. In the example just given, the extension is BNK. The expression *run bank* is also useful. A *run bank* is the combination of three files, the .VAM, the .BNK, and .IND files associated with a run of the model. All three of these files should have the same root name, and that root name is the root name of the run bank.

The words RESULT, START, EXOG, MACFIXES and VECFIXES will now be used as names of variables. The values of these variables are specified by the content of the edit boxes just to the left of these names on the form. The START variable should be given the root name of the run bank which the model begins its calculations. The first time the model is run, the START run bank will have been created by a *G7*-only version of the model. Once a base simulation has been established with a run named, say, BASE, this run bank can be used as the START run bank for subsequent runs of the model.

The RESULT field should be filled in with the root name of the run bank which will be created by the run of the model. Examples could be “Base” or “Boom” or “Crash”. Normally, the START run bank is copied to the RESULT run bank before any calculations are done. But if START and RESULT are the same, no copy can be made – a file cannot be copied onto itself – and the model begins its calculations from the run bank specified by these two names.

The next variable to be supplied by the user is EXOG, the root name of a file having the extension .XG. This is a file of *G7 commands* which change the *exogenous* variables in the RESULT run bank. These changes can include both exogenous macro variables such as population or money supply and exogenous variables in the VAM file, such as input-output matrices.

The next variable is MACFIXES. Just as macro models need to be manipulated by fixes on their behavioral equations, so too do multisectoral models. The fixes are specified in ways that are similar to those of macromodels. The details are supplied later in this chapter. For the moment, it is enough to note that the specifications should be put in a file having the extension *.mfx*. The root name might be “Base”, “Boom”, “Crash” or any other descriptive word, but not *Macfixes*. That root name should be typed into the MACFIXES field, and a program called *Macfixer* is called to prepare the fixes for use by the model. If the word “none” (without the quotation marks) is typed there, *Macfixer* is not called, and no macro fixes are used.

Besides the fixes on macrovariables, there may be fixes on vectors. As with macro equation fixes,

the details are specified later in this chapter. The specifications should be in a file with the extension *.vfx* and the root name of this file should be typed into the VECFIXES field It should NOT be *VecFixes*. These fixes are processed by a program called *Fixer*. If the word "none" is in the VECFIXES field, *Fixer* is not called and the simulation program does not look for vector fixes.

If the box "Use all data" is checked, all known values of endogenous macrovariables will be used. It should not be checked if it is desired to test the model inside the period of known historical values of endogenous macrovariables.

The type of run will normally be "deterministic". Optimization similar to that for macro models is available. Stochastic simulation is not presently (2011) available, but should not be especially difficult to program for macro equations.

Sometimes, when a model is not behaving correctly, it is useful to put in debugging printout. Often the problem is not visible until the model has run several years. Only then is the printout desired. The "Debug start year" field provides a way to supply the year in which such printing starts. If it is not used in the model, it may be given a value far out into the future, as has been done in the example.

When this form has been completed, just click the OK button, and the model will be run.

If all goes well, *that is all you need to know*. But if you would like to know more details about what happens when you click OK, here are the basics.

The five fields across the top of the form and the two panes at the bottom are used to create a file called DYME.CFG. The box below shows what it looks like for a base run of *Tiny* with no vector fixes.

```
Title of run        ;Demonstration of the Tiny Model
Start year          ;1995
Finish year         ;2010
Start MacEq yr      ;2003
Discrepancy yr      ;1995
Use all data?       ;yes
VecFix file         ;none
MacroFix file       ;Macfixes
Vam file            ;base
G bank              ;base
Debug start yr      ;2100
Max iterations      ;100
Optimization specification file; none
Number of random draws; 0
Additive random errors; no
Random coefficients; no
```

The five edit controls in the middle of the Interdyme Run Form are used to create an "add" command with arguments to the *G7* command processor. The command is:

```
add run.add <RESULT> <START> <EXOG> <MACFIXES> <VECFIXES>
```

where the values of the variables <RESULT>, <START>, etc., are taken from the form. All but the second of these names can easily be the same. For example, a base forecast might be made, starting from the historical simulation bank by the following command to *G7*:

```
add run.add base hist base base base
```

In fact, this way of working is to be recommended, for then the result of the run is a set of files which all have the same root name. In the case of the example, they would be BASE.VAM, BASE.BNK, BASE.IND, BASE.XG, and BASE.MFX. As noted, either or both of MACFIXES and VECFIXES can be "none".

The real question is now "What is the RUN.ADD file, and what does it do?" It is a simple text file that needs to be in the directory with the model. It and two helper batch files COPYTOTEMP.BAT and DYMERUNNER.BAT, also need to be in the same directory. They are shown in the boxes below.

The RUN.ADD File

```
# run.add - The arguments to this file are:
#   %1 <RESULT>, %2 <START>, %3 <EXOG>, %4 <MACFIXES>, %5 <VECFIXES>
dos CopyToTmp %2

# Revise macro bank and vam file with EXOG.XG
wsb tmp
vam tmp b
dvam b
add %3.xg
wsb ws
close b

# Tap Enter to run the model
pause

dos DymeRunner %1 %2 %3 %4 %5
```

The CopyToTemp.bat File

```
copy %1.ind tmp.ind
copy %1.bnk tmp.bnk
copy %1.vam tmp.vam
```

RUN.ADD first uses the COPYTOTEMP.BAT file to copy the <START> run bank to a temporary run bank whose components are tmp.ind, tmp.bnk and tmp.vam. Then it makes the tmp bank the G workspace bank and tmp.vam the default vam file. Then it causes *G7* to use the commands and data in EXOG.XG to update all the exogenous variables, vectors and matrices. (Remember, EXOG here is a variable which will have whatever value you gave it in the Interdyme Run Form.) With all the exogenous variables now stored in the tmp run bank, that bank is now freed by *G7* (the "wsb ws" and "close b" lines) so that they can be opened by the DYME.EXE program which runs the model.

RUN.ADD then executes the DYMERUNNER.BAT DOS batch file. If there are vector fixes, they are processed by the *Fixer* program to form the VECFIXES.FIN file. If there are macrofixes, they are processed by *MacFixer* to create the Macfixes G bank used by the Interdyme program. Note: the name of the file that specifies your macro fixes should have the extension *.mfx*, but should not be

MACFIXES.MFX. Likewise, the file that specifies the vector fixes should have the extension *.vfx*, but should NOT be VECFIXES.VFX.

The DymeRunner.bat File

```
rem DymeRunner.bat
rem The arguments here have the same meaning as in run.add.
rem If there are no Vector fixes, skip to Macro fixes.
echo %1 %2 %3 %4 %5
if %5 == none goto macrofixes
rem FAILURE
rem Here when there are Vector fixes
copy %5.vfx VecFixes.vfx
zap
fixer
copy VecFixes.chk %1.vck
copy VecFixes.vfx %1.vfx
:macrofixes
if %4 == none goto runmodel
copy %4.mfx MacFixes.mfx
macfixer
copy MacFixes.chk %1.mck
copy MacFixes.mfx %1.mfx
:runmodel
dyme

echo Dyme has finished

copy tmp.* %1.*
```

With all the exogenous data and fixes read for its use, the Interdyme model is executed by the simple command "dyme". When it finishes, the tmp files are all copied to files with corresponding extensions, but with the value of the RESULT variable as their root name. Similarly, after execution of *Fixer* and *MacFixer*, their input and check files are copied to RESULT.mfx, RESULT.mck (the check file), RESULT.vfx, and RESULT.vck (the check file from *Fixer*). If EXOG is the same as RESULT, then all of the inputs and outputs of the run will have the same root name, a fact which makes it easy to keep track of what goes into and comes out of each run of the model.

If all these steps seem a bit complicated, remember that it only takes one click on the OK button to execute them all. Note also that a record of all the input files is made with the value of RESULT as their root name.

The MACFIXER.CFG file is assumed to be:

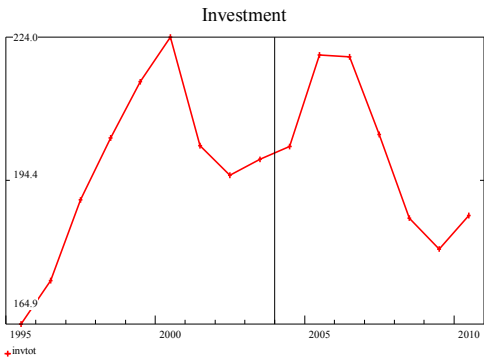
```
Input fix file      ;MacFixes.mfx
Output fixes bank   ;MacFixes
Model G bank        ;tmp
Output check file   ;MacFixer.chk
```

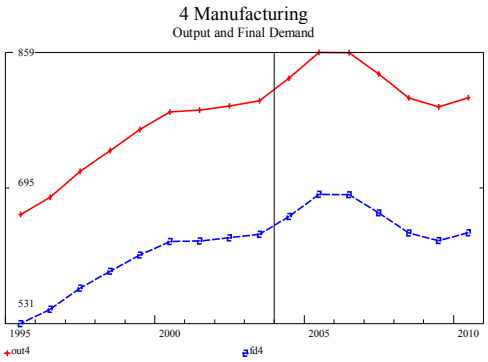
The FIXER.CFG (for *Fixer*) is assumed to be:

Input fix file	;VecFixes.vfx
Fix index (.fin) output file	;vecfixes
VAM reference	;tmp
Text check file	;fixer.chk

While these files can be changed by the user, doing so will probably cause the Interdyme Run Form to stop working. The flexibility gained by changing these files is unlikely to be worth the confusion it will probably cause.

Here are the graphs of total investment and the output of manufacturing for this simplest version (model 1) of the *Tiny* Interdyme model.





The results are clearly looking more like model results than they did with all demand exogenous.

Lest you get lost in all the details, let's review what you actually have to do to run *Tiny* Model 1. You start *G7* in the \Tiny\Model directory and follow the steps in the box on the next page.

In *G7:*

add tiny.pre (creates HIST.VAM file with historical data and project the coefficients into future periods.)

add invtot.reg (estimate regression equations)

(Here you could edit MODEL.CPP to make changes for new features)

Click **Model | Run IdBuild and compile model**

Edit EXOGALL.REG to set tdates 2003 2010 and set limits to 1995 2003 2010

In *G7:*

add exogall.reg (to generate forecasts of exogenous variables; edit the forecasts if desired)

Edit RUN.XOG to make desired changes in the exogenous forecasts; add commands to read exogenous values of matrices and vectors, and save the result as BASE.XG.

Click **Model | Run Dyme Model** and fill in the blanks as desired and click OK.

Back in *G7*, to graph the results, type

```
gdates 1995 2003 2010
vam base b
fadd graphs.fad sectors.ttl
```

To make tables, click **Model | Express Tables**

We will add a number of features to *Tiny* to illustrate things you can do with a model of a real economy. But before we go further, it is time for a few exercises.

Exercises

1. Make alternative forecasts for some of the exogenous variables in *Tiny*, run the model with them, graph the results, and show them in tables.

2. Estimate a regression equation for total imports as a function of GDP, put it into the model, run the model and compare graphically the results with those without your function.

More on Interdyme Programming

Before you can build a *Tiny*-like model for your own imaginary or real economy, we have to deal with one more basic question: How is the Interdyme program connected with the VAM file? How does it know, for example, that there are vectors in *Tiny*'s VAM file named *fd*, *pce*, *gov*, *inv* and so on? The answer is that the model builder has to indicate to the model the names of these vectors and matrices. The VAM file is then used to determine their dimension. This identification has to be done in two steps. First, there is a file called USER.H; the one for *Tiny* is shown in the box below.

The USER.H File for *Tiny* Model 1

```
// USER.H -- Put here any includes that refer to the user model, per se.
//   This version is for Tiny, Model 1

// These names will be read From DYME.CFG and opening screen:
GLOBAL char RunTitle[80],CfgFileName[80],VamFileName[80],GbankName[80],
   VecFixFileName[80],MacFixFileName[80];

GLOBAL char* outfix; // Determines how Seidel will determine output

// Vector declaration:
GLOBAL Vector out, pce, gov, inv, ex, im, fd,
   dep, lab, cap, ind, va, depc, labc, capc,
   indc, pcec, invc, govc, exc, imc, x, y, fix;

// Matrix declaration
GLOBAL Matrix AM;

// Integer Vector that gives the triangulation order of sectors
GLOBAL IVector triang;
```

All the variables declared in USER.H are “global”, C jargon for variables which can be accessed from anywhee in the program. This fact is indicated by the word GLOBAL in front of these variables. To explain exactly what is going on here requires some explanation, which you can skip if you are familiar with C and C++, or don't want to bother with the details of exactly how this GLOBAL keyword works.

Unlike Basic and Fortran, C and C++ are strictly “typed” languages. That is, the nature of every variable must be declared before it can be used. Typical declarations are:

```
char sex;       // char is a character variable
int zip;        // int is an integer
float height, weight;   // float is a real number
char name[40];  // this is an array of characters
```

The variable *sex* is a single character, presumably 'M' or 'F'; the variable *zip* is an integer, something like 20742; *height* and *weight* are floating point numbers, that is, numbers that potentially have a fractional component like 72.5 or 217.8. The variable *name* is a string of up to 40 characters, something like “Thomas”. If a variable is declared inside a function, it is local to that function.

Other functions know nothing about it. But a variable declared outside of all functions, typically near the top of a file containing C code, is global and can be accessed by all functions in that file. A large program such as Interdyme usually consists of a number of files with names ending in .CPP; each of these files is called a module, and is compiled separately. If some variable, say *name*, is to be accessed in several different modules, then it must be declared globally in all the modules where it is accessed. But one and only one of the modules should actually make space for it. Which one? To answer this question, the program should mark all the declarations which are not to make space as **extern**, like this:

```
extern char name[40];
```

In one and only one module should appear the simple declaration:

```
char name[40];
```

That is the module where the space is allocated for the variable. When the compiled modules are "linked" to form one whole executable program, the references to the variable in all the modules where it was external are made to point to the space allocated by the one module where it was not external.

It could potentially be quite a nuisance to remember to mark all but one declaration as **extern**. That is where the word "GLOBAL" comes in handy. It is not a standard C keyword but is used in Interdyme, *G7,* and many other programs. The declarations of all global variables are put into "header" files like USER.H, and are marked GLOBAL. These header files are then "included" into all the modules where they are relevant by a compiler directive like

```
#include "user.h"
```

In all but one module, this directive is preceded by another:

```
#define GLOBAL extern
```

In these modules, the compiler will replace "GLOBAL" by "extern" before compiling. In that one and only one other module, namely DYME.CPP in Interdyme, the "#include" is preceded by

```
#define GLOBAL
```

which defines GLOBAL to be nothing, so that in this module the "extern" is omitted and space is made for the global variables.

When a vector or matrix is declared locally, it can be fully "constructed", that is, space allocated for its elements. But there is a problem with declaring an object like a vector globally; since the declaration is outside of any function, no computing can be done. To find out how much space to allocate for the array of numbers in the vector, the VAM file has to be read, but reading the VAM file is computing, so what needs to be done? The answer is that once computing has begun, we must *resize* all the global matrices and vectors. That is, we must read the VAM file, find out how big the matrix or vector is, grab enough memory to hold it, and stick the pointer to that memory into the space saved for the pointer by the global declaration.

That may seem complicated to understand, but it is easy to do. Look at MODEL.CPP for TINY with the *G7* or other text editor. The lines concerned with resizing are the following:

```
// Resize Vectors
```

```
out.r("out"); pce.r("pce"); gov.r("gov");
inv.r("inv"); ex.r("ex"); im.r("im"); fd.r("fd");
dep.r("dep"); lab.r("lab"); cap.r("cap"); ind.r("ind");
depc.r("depc"); labc.r("labc"); capc.r("capc"); indc.r("indc");
pcec.r("pcec"); invc.r("invc"); govc.r("govc"); exc.r("exc");
imc.r("imc"); x.r("x"); y.r("y");

 // Resize Matrices
 AM.r("AM");
```

The "resize" function or method is abbreviated to just .r . The argument to the resize function is the name of the vector or matrix in the VAM file. Preparing these lines for USER.H and MODEL.CPP from the VAM.CFG is a lot of rather mechanical, error-prone work. However, when you gave *G7* the command:

vamcreate vam.cfg hist

it also wrote two files, VAM.GLB and VAM.RSZ, as follows

The VAM.GLB file:

```
GLOBAL Vector out, pce, gov, inv, ex, im, fd,
  dep, lab, cap, ind, depc, labc, capc,
  indc, pcec, invc, govc, exc, imc, x,
  y;
GLOBAL Matrix FM, AM, LINV;
GLOBAL Matrix OUTlag;
```

The VAM.RSZ file

```
out.r("out"); pce.r("pce"); gov.r("gov");
inv.r("inv"); ex.r("ex"); im.r("im"); fd.r("fd");
dep.r("dep"); lab.r("lab"); cap.r("cap"); ind.r("ind");
depc.r("depc"); labc.r("labc"); capc.r("capc"); indc.r("indc");
pcec.r("pcec"); invc.r("invc"); govc.r("govc"); exc.r("exc");
imc.r("imc"); x.r("x"); y.r("y");
FM.r("FM"); AM.r("AM"); LINV.r("LINV");
OUTlag.r("out");
```

They are in the \tiny\model directory. You will notice a striking similarity to the corresponding portions of the USER.H and MODEL.CPP files. All you have to do is bring these two files written by *G7* into USER.H and MODEL.CPP , respectively, with the *G7* or other text editor. For the model, I removed the *FM* and *LINV* matrices, for they will not be used in the Interdyme model. Likewise, I removed *OUTlag* matrix, which is used to store the lagged values of the *out* vector, because the lagged values are not yet in use.

The rest of the *loop()* function should be regarded as the standard form, which the user of Interdyme should have no need to change. The *spin()* function, which we have looked at in detail, is where the changes have to be made for different models.

Exercise

3. Build a TINY- like model for your imaginary economy or for a real economy for which you have data readily available.

16.2. Matrix Tools in Interdyme

Here is a quick overview of the actions, operators, and functions available in Interdyme. Some of them we have seen, but others were not needed in the TINY example. You need not learn the exact syntax of each of them; just make a mental note of the possibilities.

If *A* is a Matrix or Vector and *k* is a scalar (a float in C terms), then

*k***A*	multiplies each element of *A* by *k*.
A/*k*	divides each element of *A* by *k*.

If *A* and *B* are both Matrices or both Vectors of the same dimension, then

A + *B*	gives the matrix or vector sum
A - *B*	gives the matrix or vector difference
ebemul(*A*,*B*)	gives the element-by-element product
ebediv(*A*,*B*)	gives the element-by-element quotient, the elements of *A* being divided by the corresponding elements of *B*. If, an element of *B* is zero, the corresponding element of A is returned in that position.

If *A* has the same number of columns as *B* has rows, then

*A***B*	gives the matrix product.

If *A* and *B* have the same number of columns, then

A/B	gives the same thing as ~A*B that is, the transpose of A multiplied by B, but without actually forming the transpose of A. (Think of the / as being a ' to denote transposing the matrix.)

Interdyme understands parentheses. If all the dimensions are appropriate, the following is an acceptable statement:

A =k* (B+C+D)*(E + F + G*(H+I));

If *x* and *y* are both Vectors with the same number of elements:

dot(*x*,*y*)	gives the inner product as a float.

If *A* is a Matrix and *x* is a Vector with the same number of elements as *A* has columns,

A%x	gives the result of converting *x* to a diagonal matrix and then post-multiplying *A* by this diagonal matrix. Essentially, it multiplies each column of *A* by the corresponding element of *x*.

If A is a Matrix and x is a Vector with the same number of elements as A has rows,

x%A	gives the result of converting *x* to a diagonal matrix and then premultiplying *A* by this diagonal matrix. Said perhaps more simply, it multiplies each row of *A* by the corresponding element of *x*.

The first of these % operations is useful in computing a flow matrix from a coefficient matrix and a vector of outputs. The second can compute a flow matrix in current prices from one in constant prices.

For a Matrix *A*, Vector *v*, float *z*, and int *k*,

v.set(z)	sets all elements of Vector *v* to *z*.
A.set(z)	sets all elements of Matrix *A* to *z*.
pulloutcol(v, A, k)	pulls column *k* of *A* into *v*.
putincol(v, A, k)	puts v into column k of A.
pulloutrow(v, A, k)	pulls row *k* of *A* into *v*.
putinrow(v, A, k)	puts *v* in row *k* of *A*.
v = colsum(A)	puts the column sums of *A* into the vector *v*.
v = rowsum(A)	puts the row sums of *A* into the vector *v*.
z = v.sum()	puts the sum of the elements of *v* into *z*.
v.First()	gives the number of the first row of *v* if *v* is column and vice versa.

If *A* is a square, non-singular matrix,

!A	gives the inverse of *A*.
A.invert(i,j)	transforms *A* into its inverse by Gauss-Jordan pivoting. The pivot operations start in row *i* and stop when the pivot has been in row *j*. If these arguments are omitted, the pivoting starts in the first row and continues through the last, to produce the true inverse.

The difference here is that !A does not change *A* but creates a new matrix for the inverse while A.invert() transforms *A* into its inverse. Thus, if memory space is scarce, the invert action may be preferable. The algorithm in both cases is Gauss-Jordan pivoting with no niceties. Don't trust it if your matrix poses any problems for inversion.

If A a is either a Vector or Matrix object, then

~A	gives the transpose of *A*.
A.rows()	gives the number of rows as an integer.
A.columns()	gives the number of columns as an integer.
A.firstrow()	gives the number of the first row as an integer.
A.lastrow()	gives the number of the last row as an integer.
A.firstcolumn()	gives the number of the first column as an integer.
A.lastcolumn()	gives the number of the last column as an integer.

If A is a square Matrix and q and f are Vectors of the appropriate dimension, the equation

```
q = Aq +f;
```

can be solved by the Seidel iterative method (if it converges) by the function

```
Seidel(A, q, f, triang, toler);
```

where *triang* is an array of integers giving the order in which the rows of *A* should be selected in the Seidel process, and *toler* is a float giving the tolerance which is accepted in the iterative solution. Similarly, the equation

```
p = pA + v;
```

can be solved by

```
PSeidel(A, p, v, triang, toler);
```

If you need a temporary Matrix *A* or Vector *B* is not in the VAM file, you can declare it locally in the function where it is needed by:

```
Matrix A(n,m);
Vector B(n);
```

were *n* is the number of rows and *m* is the number of columns.

In the process of debugging a program, it is sometimes useful to display a Matrix or Vector *A* on the screen. To do so, use

```
A.Display("message", fieldwidth, decimals);
```

To write a Matrix *A* to a file use

```
writemat(A, filename,fieldwidth, decimals);
```

To write a Vector *A* to a file use

```
writevec(A, filename, fieldwidth, decimals);
```

For completeness, we mention a function which will be explained in following sections. A Matrix *A* can be balanced to have the row sums given by Vector *a* and column sums given by Vector b by the function

```
int ras(A, a, b)
```

If the sum of the elements of *a* and *b* are not equal, the user is required to pick which governs.

Finally, we should mention that all of these matrix routines are available in a matrix package called BUMP (Beginner's Understandable Matrix Package) which can be used in C++ independently of Interdyme. The code of BUMP is carefully explained so that beginners of C++ can learn how to write such functions. Understanding how it works, however, is not necessary for using the functions in Interdyme any more than it is necessary to know how Excel is programmed in order to use it.

16.3. Vector Elements in Regression Equations

This section will use *IdBuild* to develop consumption equations for personal consumption, even though the left-hand side variables are vector elements. This version of the *Tiny* model is Model 2.

So far, we have used only one behavioral, regression-estimated equation, the one for investment. In this section, we will add regression equations for all components of Personal consumption

expenditures. In the process, we will introduce several new techniques. So far, we have used only macrovariables in regressions. Now we will see how to use elements of vectors in regression. To ensure that the total of the predicted values stands in a reasonable relation to disposable income, we will need to learn about static vectors and apply some vector arithmetic.

In the Tiny\Model directory, you will find the file PCE.DAT shown below.

PCE.DAT

```
# vamcr vam.cfg test
# vam test b
# dvam b
vmatdat r 1 9   1 8  5
pce 1995 1996 1997 1998 1999 2000 2001 2002 2003
#"  Date" "   pce1$" "   pce2$" "   pce3$" "   pce4$" "   pce5$" "   pce6$" "   pce7$" pce8$
1995       14.169    1.908    76.759   283.944   288.323   109.289   443.041   0.0
1996       14.101    1.917    78.365   299.580   295.352   115.091   446.638   0.0
1997       14.127    1.934    76.941   317.095   304.522   121.077   454.966   0.0
1998       14.228    1.972    77.331   343.489   317.501   123.079   466.458   0.0
1999       14.519    2.016    77.277   375.369   334.601   126.768   478.526   0.0
2000       15.000    2.000    80.000   400.000   350.000   130.000   500.000   0.0
2001       14.947    2.001    77.527   407.622   353.149   127.153   506.602   0.0
2002       14.821    1.985    77.343   419.457   356.482   121.248   508.665   0.0
2003       14.974    1.928    75.216   435.677   364.071   115.282   507.851   0.0
```

Open *G7*, assign the HIST.VAM file as bank b, and make it the default vam file. Then introduce the data in the PCE.DAT with an "add pce.dat"statement, and check that it has been correctly read with the "show" command.

The PCE.REG File

```
lim 1995 2003 2010
catch pce.cat
save  pce.sav
f dpdis = pdisinc - pdisinc[1]
ti PCE on Agriculture
r pce1 = pdisinc, dpdis
gr *

. . . .

ti PCE on Services
r pce7= pdisinc, dpdis
gr *
save off
catch off
```

From this data, we now want to estimate simple consumption equations by regressing each component of the Personal consumption expenditure vector, *pce*, on personal disposable income (*pdisinc*) and its first difference (*dpdis*). You will also find in the \tiny\model directory the PCE.REG file shown, in abbreviated form, on the right. (The . . . . show where there are similar triplets of commands for regressions for sectors 2, 3, 4, and 5 in the full file on the disk.) You can now execute this command file in *G7* either by "add pce.reg" or by opening the file in the editor and clicking "Run". You will see that *G7* has no problem figuring out that pce1, pce2, ..., pce7 are elements of the pce vector in the default VAM bank.

IdBuild, however, is not so clever. It knows nothing about the VAM bank. In response to the command "iadd pce.sav", it will give a number of error messages such as "Cannot find pce1." The solution, however, is simple. We just need to tell *IdBuild* that pce is a vector. We do so with the command

```
isvector pce
```

in the MASTER file. Here is the complete MASTER file for the TINY model with the PCE equations. The new material is in bold type.

Master File for TINY with PCE Equations

```
isvector pce
iadd pce.sav
isvector clear
iadd invtot.sav
iadd account.sav
iadd pseudo.sav
end
```

From it, *IdBuild* will produce a HEART.CPP file with the section shown in the box below for the PCE equations. Note first that *pce* Vector must be passed to the function *pcef()*; Secondly, note that the left side of equation stores the value computed by the equation directly into the appropriate element of the *pce* vector. There is no provision here for the automatic application of fixes or rho adjustments. How fixes or adjustments may be applied we will see in a later section. Finally, back in the Master file above, note the "isvector clear" command in the third line. If it were not there, *IdBuild* would write the following *invtotf()* and *accountf()* functions so that they also had to be passed – quite unnecessarily – the *pce* vector.

Personal Consumption Equations in the HEART.CPP File

```
void pcef(Vector& pce)
{
dpdis[t]= pdisinc[t]- pdisinc[t-1];
/*  PCE on Agriculture */
pce[1] = coef[0][0]+coef[0][1]*pdisinc[t]+coef[0][2]*dpdis[t];
/*  PCE on Mining */
pce[2] = coef[1][0]+coef[1][1]*pdisinc[t]+coef[1][2]*dpdis[t];
/*  PCE on Gas and Electricity */
pce[3] = coef[2][0]+coef[2][1]*pdisinc[t]+coef[2][2]*dpdis[t];
/*  PCE on Manufacturing */
pce[4] = +coef[3][0]+coef[3][1]*pdisinc[t]+coef[3][2]*dpdis[t];
/*  PCE on Commerce */
pce[5] = +coef[4][0]+coef[4][1]*pdisinc[t]+coef[4][2]*dpdis[t];
/*  PCE on Transport */
pce[6] = coef[5][0]+coef[5][1]*pdisinc[t]+coef[5][2]*dpdis[t];
/*  PCE on Services */
pce[7] = coef[6][0]+coef[6][1]*pdisinc[t]+coef[6][2]*dpdis[t];
}
```

Equations for Personal consumption expenditures (PCE) need a property not required of the equations for most other variables, namely, they must add up properly. More precisely, the sum of the predicted values from the PCE equations plus Personal savings must equal Personal disposable income. We could, of course, just let savings be a residual, but it is too important for the macroeconomic properties of the model to be treated so casually. So we generally want to have an equation for Personal savings – and for any other items in the difference between total PCE and disposable income, of which there are none in *Tiny*. The best way to achieve this equality is to add up the predicted pieces, compare the sum with the desired total, and spread the difference over the components in some pre-determined shares. The shares we have used are proportional to the income coefficients of the various regression equations. The shares were chosen in this way because the discrepancy between the sum of the predicted values and the desired total can be thought of as a

little more or little less income to be divided among the various goods purchased. Sometimes we have used the standard error of estimate of the different equations, on the grounds that the changes should be greatest where the uncertainty about the right value is greatest.

The mechanics of how the discrepancy is allocated is, however, independent of how the shares were determined. We create a text file, which we shall call PCESPREAD.DAT, which has the shares we want to use, however we got them. They should, of course, sum to 1.0. The box to the right shows this file for *Tiny*. In the file USER.H, we need to add at the end the line

```
GLOBAL Vector PCESpread;
```

to declare globally the Vector which will hold the shares for spreading.

The PCESPREAD.DAT file

0.003175
0.000236
0.000000
0.481880
0.248311
0.036523
0.229874
0.000000

To make use of the new equations, we need to make a few changes in MODEL.CPP. On the disk, the modified file is called MODEL2.CPP; copy it to MODEL.CPP to make the changes take effect. The code snippets below show excerpts from the new MODEL2.CPP.

The first order of new business in the *loop()* function is to resize the *PCESpread* vector which has been declared globally and then to read in its data. That job is accomplished by the lines

```
// Resize and read the vector for spreading the PCE discrepancy.
PCESpread.resize(NSEC);
PCESpread.ReadA("PCESpread.dat");
```

Note that the resizing is done by the *resize()* function of the Vector, not by the *r()* function. The *r()* function looks in the VAM file to find the dimension of the Vector, so it won't work for the *PCESpread* Vector, because it is not in the VAM file. The *resize()* function is given the size directly as its argument. In *Tiny*, NSEC has already been given the value of 8.

The predicted values of the elements of the *pce* vector are computed from the regression equations by the function call

```
pcef(pce);
```

The lines

```
// Sum up the calculated PCE elements
pcesum = pce.sum();
pcediscrep = pcetot[t] - pcesum;
// printf("\npcediscrep = %10.2f\n",pcediscrep);
// Spread discrepancy by the proportions of PCESpread vector.
pce = pce + pcediscrep*PCESpread;
```

sum up the calculated PCE elements, subtract the sum from the desired total, and spread the discrepancy among the sectors in the proportions given by the *PCESpread* vector. The line

```
// printf("\npcediscrep = %10.2f\n",pcediscrep);
```

is now just a comment without effect on the operation of the program. If the // at the beginning were removed, it would print the discrepancy at each pass through the loop. It can be used it to check that the discrepancies are indeed small. Instead, however, of totally removing it, it is left as a comment

which may be a useful illustration of how to include such debugging printing.

You should now estimate the PCE equations, make the required changes in USER.H and MODEL.CPP, do Model | Run *IdBuild* and compile model, then do Model | Run dyme and graph the resulting elements of the pce vector.

To review, the two new techniques introduced in this section were the "isvector" command to *IdBuild* and the use of a static vector not in the VAM file.

The "isvector" command, by the way, also allows vector elements to be used on the right-hand side of regression equations. For example, in a more detailed model than *Tiny*, the output of the Railroad industry may be used in the equation for Railroad construction.

16.4. Systems of Detached-Coefficient Equations

In this section, we'll develop the version of *Tiny* we call Model 3, and you can copy MODEL3.CPP to MODEL.CPP. This section introduces the use of "detached-coefficient" equations.

The PCEEQN.REG File

```
catch pce.cat
lim 1995 2003
vam hist b
dvam b
f dpdis = pdisinc - pdisinc[1]

punch pce.eqn 7 4 2003

ti PCE on Agriculture
r pce1 = pdisinc, dpdis
ipch pce 1 L
gr *

. . . .

ti PCE on Services
r pce7= pdisinc, dpdis
ipch pce  7 L
gr *

punch off

catch off
```

The use of the "isvector" command in *IdBuild* is perfectly satisfactory for elements of vectors on the right-hand side of the regression equation; but for variables on the left-hand side, it has its limits. In the first place, automatic rho-adjustment is not possible. Secondly, it cannot be used to pass the whole system of import equations to the Seidel routine so that imports dependent on domestic demand can be calculated simultaneously with product outputs. Nor can it be used to pass a whole system of consumption functions, such as the PADS system (covered in chapter 20) to a function to compute predicted values as a fairly complicated function of the parameters. All of the problems are overcome in through the use of a system of "detached-coefficient" equations. Actually, this method is older than the "isvector" method and goes back to models developed in the early 1960's. The "detached-coefficient" name comes from the fact that the regression equations are "detached" from the variables names and stored in a separate file by *G7*. The added complexity of this method, however, is that program has to be written to interpret the equations. For Seidel with import equations and for PADS, however, the code is already written and part of the Interdyme system.

In this section, we will see how to use the detached-coefficient method for the PCE equations we have already estimated. We begin from the PCEEQN.REG file shown in the box to the right. It is a command file for *G7*; as usual, the four dots indicate the omission for printing here of a repetitive portion of the text of the file. There are two new commands, "punch" and "ipch". The "punch" command

```
punch pce.eqn 7 4 2003
```

opens a file to be called PCE.EQN to receive the regression coefficients. There will be 7 equations with up to 4 coefficients each. The last year of data used in estimating them will be 2003. After each equation is estimated there is an "ipch" command such as

```
ipch pce 1 L
```

which writes to the open "punch" file the rho and the regression coefficients for the most recently estimated regression equation. In the file, it will be labeled as the equation for the pce vector, element 1. The "L" will be written to file to indicate the type of equation. It could be any one letter or number; we will write the program to interpret these letters correctly. The L in this particular case stands for "Linear". The file ends with the command

```
punch off
```

to close the file to which *G7* has been writing the regression results. The name of the command, "punch", refers to punching the results into cards and indicates just how ancient this method is. The cards are gone, but the name works fine. The PCE.EQN file produced by *G7* from the PCEEQN.REG file shown above is shown in the box below.

The PCE.EQN File

```
7 4 2003
pce 1 L 3
1 2 3
    0.280107      10.1751   0.00312443  -0.00190926
pce 2 L 3
1 2 3
    0.456294      1.62189  0.000232365  0.000253049
pce 3 L 3
1 2 3
    0.150868      76.9213 -6.45766e-06    0.0127951
pce 4 L 3
1 2 3
    0.214085     -303.312     0.474066    -0.155993
pce 5 L 3
1 2 3
    0.268659     -14.8633     0.244285   -0.0810024
pce 6 L 3
1 2 3
    0.572774      66.8459    0.0359306    0.0773437
pce 7 L 3
1 2 3
  -0.0435048      164.664     0.226147    -0.179012
```

The three numbers on the first line are taken from the "punch" command that opened the file and have already been explained. For each equation there are then three lines. The first gives, somewhat redundantly, the name of the vector of the dependent variable, the number of the element in the vector, the type of equation, and the number of regression coefficients for it. The second line specifies which coefficients will be given in the next line, and the third line gives first the rho estimated for the equation and then the regression coefficients in the order specified by the second

line.

In our case, the second line may seem unnecessary since it is both very simple and always the same. It can, however, add considerable flexibility. We might have had, for example, a more general form of regression in which there were five possible independent variables with a relative price term as the fourth and a time trend as the fifth variable. In that case, had the equation for element 7 used only the intercept and the time trend, the command to *G7* would have been

```
r pce7 = time
ipch pce 7 L 1 5
```

and in PCE.EQN the resulting lines would have been something like

```
pce 7 L 2
1 5
  -0.0435048      164.664     12.345
```

where 12.345 is the coefficient on time. Since the coefficient matrix is originally set to zero, this device allows one type of equation to be used for all variants of an equation that differ from a base type only by omitting one or more of the variables. Note that the numbers at the end of the "ipch" command are optional. If they are not specified, then *G7* assumes that they are the integers up to the number of variables in the equation.

Now with the equations estimated and stored away in the PCE.EQN file, we have to get them into the Interdyme program and tell Interdyme how to interpret the coefficients. To do so, we will use the C++ concept of a "class", namely, of a class called "Equation.". Generally, a class is a combination of data and ways of working with the data. We have already met the Matrix and Vector classes. Instances of a class are called "objects." For example, in USER.H we need to put the line

```
GLOBAL Equation pceeqn;
```

It will make the object *pceeqn* an instance of the class Equation. Then in MODEL.CPP, we put the lines

```
// Resize, read, and store the PCE equations; reads the pce.eqn file.
pceeqn.r("pce.eqn");
```

The second of these calls the *r()* function (sometimes called method) of the *pceeqn* object to read the file PCE.EQN, claim enough space for storing the data of the PCE equations, and read the data from the file into that space. How does the program know how to claim the space and read the data? Any object of the Equation class knows how to read the data, how to dish out the coefficients of the equations and some other data to using programs, and how to apply rho adjustments to the predicted values. It does NOT know how to use the coefficients to compute the predicted values. The user, you, must write those instructions. We will see how in a moment. First, here is an except from MODEL.CPP which shows the new code for reading the equation data in context in the *loop()* function.

```
// Resize and read the vecor for spreading the PCE discrepancy.
PCESpread.resize(NSEC);
PCESpread.ReadA("PCESpread.dat");

// Resize, read, and store the PCE equations; reads the pce.eqn file.
pceeqn.r("pce.eqn");
```

```
// pceeqn.rhostart is read as the last year used in estimating the PCE equations.
// We want it to be the year in which to start the rhoadjustment.  If we are
// doing a historical simulation, that should MacEqStartDate if it is before the
// date recorded in the pce.eqn file.
if(pceeqn.rhostart > MacEqStartDate) pceeqn.rhostart = MacEqStartDate;
```

In the *spin()* function, the new equations are called to calculate the *pce* vector by the line

```
pcefunc();
```

shown in context in the lines below.

```
// Compute the estimated PCE equations with equation system
if(t>= MacEqStartDate){
   pcefunc();
   // Sum up the calculated PCE elements
   pcesum = pce.sum();
   pcediscrep = pcetot[t] - pcesum;
   // printf("\npcediscrep = %10.2f\n",pcediscrep);
   // Spread the discrepancy by the proportions of the PCESpread vector.
   pce = pce + pcediscrep*PCESpread;
   invtotf();
   }
inv = invtot[t]*invc;
```

. . . .

As already noted, the function *pcefunc()* that calculates the predicted values must be written by the user. Consequently, it is in MODEL.CPP, and we had better have a good look at it. It is shown in full in the box below.

The first thing to be explained is the difference between the equation number, for which the program uses the variable *i* and the sector number, for which it uses the variable *j* . In the case of *Tiny*, the PCE vector has 8 elements, one for each sector, but there are only 7 equations, because one sector (sector 8) is always zero. In larger models or for other dependent variables, the difference between the number of sectors and the number of equations can be much greater. The equations are read and stored in the order estimated, which is not necessarily the order of the sector numbers. For each equation, however, the corresponding sector number is stored in the *sec* variable Thus,

```
j = pceeqn.sec(i);
```

will put into *j* the sector number corresponding to equation *i* . The single character indicating the type of equation is stored in the *type* variable. Thus

```
which = pceeqn.type(i);
```

will put into the char variable *which* the type of equation *i* . In our case so far, it will be the letter L . Finally, the regression coefficients of equation *i* are given by *pceeqn[i][1]*, *pceeqn[i][2]*, *pceeqn[i][3]*, and so on. In our case, these are the constant term, the coefficient of *pdisinc* and *dpdis*, respectively. (With this explanation, the code down to as far as the comment

```
// Apply rho-adjustment
```

Function to Calculate PCE from Detached-Coefficient Equations

```
// pcefunc() -- PCE functions for TINY
int pcefunc(){
    int n,i,j,t1;
    float cons;
    char which;

    n = pceeqn.neq; // Number of equations
    // pdisinc is personal disposable income. It is a global macro variable.
    // Compute variables used in several equations.
    dpdis[t] = pdisinc[t] - pdisinc[t-1];

    // Loop over the equations
    for(i = 1; i <= n; i++){
       j = pceeqn.sec(i); // j is the sector number of this equation.
       which = pceeqn.type(i);  // which is the equation type
       if(which == 'L'){
          cons = pceeqn[i][1] + pceeqn[i][2]*pdisinc[t] + pceeqn[i][3]*dpdis[t];
          }
       else {
          printf("Unknown equation type %c in pcefunc, category %d.\n",
             which,i);
          tap(); //Pause so that error message can be read.
          continue;
          }
       // Apply rho adjustment
    // Note the use of i and j in the following statement.
       pce[j] = pceeqn.rhoadj(cons,pce[j],i);
       }
    pce.fix(t);
    return(n);
    }
```

should be reasonably clear.) The use of the rho-adjustment requires more explanation.

In macro models, the calculation of the initial error from which the additive rho-adjustment factor begins is simple enough. It is calculated in the first period of the run. Matters are more complicated in the multisectoral model case because not all data is equally up-to-date. We may, for example, have macroeconomic variables through 2005 but detailed PCE data only through 2003. The starting error for rho adjustments must therefore be calculated for PCE components in 2003 and used in 2004, 2005, and later years. For a macro variable, however, actual data can be used for 2004 and 2005, and the starting rho-adjustment error calculated in 2005 and used thereafter. That is why the variable *rhostart* is part of the Equation object. As read from the data file, it gives the last year of estimation, which is presumably the last year of data and therefore the last year in which the base error of the rho-adjustment process can be calculated. A further complication is that, since predicted values of one variable may depend upon calculated values of other variables, the rho-adjustment errors cannot be set until the model has converged for the period. Finally, sometimes we want to do an historical simulation and set all the rho-adjustment errors in the first period of the run. In that case, we uncheck the "Use all data" box on the screen which appears when we click Model | Run Dyme Model (see p. 53).

Bearing all that in mind, what happens when the program executes the statement

```
pce[j] = pceeqn.rhoadj(cons,pce[j],i);
```

near the end of the program shown in the box? The *rhoadj()* function of the *pceeqn* object is passed

the calculated value (*cons*), the value already in the vector (*pce[j]*), and the equation number, (*i*). Simply put, the function then figures out what it is supposed to do and does it. More specifically, if we are using all data and data is still available, it returns the actual value. If we are past the end of the data, it adds on the rho-adjusted error to the value predicted by the equation and returns that. If the signal that the model has converged (*setrho*) has been set and we are in the period in which the base error should be computed, it does so, saves it, and returns the actual value. If the model has converged, but we are already past the year for calculating the error, it adds on the error for this year to the computed value and updates the error for the next year by multiplying this year's error by the appropriate rho factor.

That leaves a question. How does the *rhoadj* function know that the model has converged? Why does it need to know that? Because only then can it calculate the starting error for the rho-adjustment in the appropriate year, or in later years, multiply this year's errors by the appropriate rho to get them ready to be added to the values predicted by the equations in the next year. The signal that the model has converged is the variable *setrho*. This *setrho* can have one of two values, 'y' for yes or 'n' for no. Near the top of the MODEL.CPP file, we see the lines

```
float pcesum, pcediscrep;
setrho = 'n';
int Iteration;
```

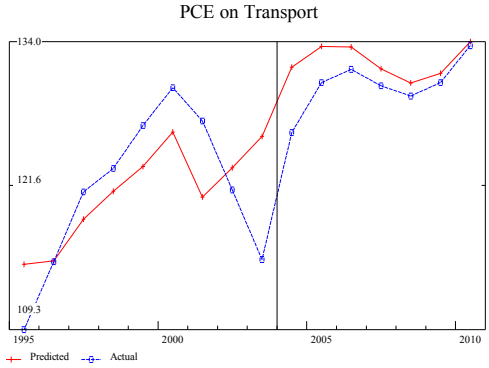
so we know it starts off as 'n'. What happens next is shown in code snippet from MODEL.CPP in the box below. The **while** loop begun in the first line continues until convergence is reached (or the limit on iterations is hit). On reaching convergence, the program breaks out of the **while** loop, sets *setrho* to 'y', calls the two functions involving rho adjustment – namely *pcefunc()* and *invtot()* – and then puts *setrho* back to 'n'.

As with macromodels, there other kinds of fixes besides rho adjustment. How to specify them will be explained in section 16.8. Here we just note that the statement

```
pce.fix(t);
```

at the end of the *pcefunc()* function will apply the fixes to be explained in that section.

The fact that the rho-adjustment is working is seen very clearly in the graph on the next page. For it, we made the VAM file BASE.VAM the default VAM file, copied the variable *pdisinc* from the BASE bank into the G workspace, and then estimated the regression in test mode, the default mode. Thus, the "actual" line in the graph is history up to 2003 and model forecast thereafter, while the "predicted" line is the prediction from the equation without rho-adjustment but made with the model-predicted values of disposable income. The way the model forecast remembers the big error of the equation in 2003 indicates clearly that the rho adjustment is working.



16.5. Import Equations

So far we have considered imports as exogenous. In fact, they are very dependent on domestic demand, so it makes sense to relate them to domestic demand and then calculate them from the estimated equations simultaneously with the calculation of output. In this section, we will see how to do exactly that. The new code is all in MODEL4.CPP, which you should copy to MODEL.CPP.

The IMPR.DAT file, shown to the right, will, when introduced in *G7* with the "add" command, provide values for the *im* vector from 1995 to 2003. Up until now, imports have been treated as negative numbers because it is convenient to do so in the input-output table. In this file however, imports are positive numbers, as they usually are in statistical sources. We will use them henceforth as positive numbers and make the required changes in our program.

The IMPR.dat File

```
vmatdat r 1 9   1 8  5
im 1995 1996 1997 1998 1999 2000 2001 2002 2003
#Date      1       2          3      4          5 6
7         8
1995    19.753   8.118  0  108.065   0 0  14.470  0
1996    19.320   8.236  0  113.841   0 0  14.795  0
1997    18.808   8.612  0  124.300   0 0  15.561  0
1998    18.114   8.619  0  130.503   0 0  16.307  0
1999    18.732   9.068  0  145.711   0 0  17.461  0
2000    20.000  10.00   0  170.000   0 0  20.000  0
2001    19.068   9.302  0  155.832   0 0  18.902  0
2002    18.992   9.122  0  156.815   0 0  18.665  0
2003    20.105   9.763  0  166.286   0 0  19.018  0
```

It is natural to suppose that imports of product i , m_i , are a linear function of domestic demand for the product, d_i , thus

$$m_i = \alpha_i + \beta_i d_i .$$

Conceptually, domestic demand for a product is the row total for that product in the input-output table without subtracting imports. Thus, conceptually, it does not depend on imports. Statistically, however, it is most easily calculated as domestic output, q_i , plus imports (as a positive number), thus:

$$d_i = q_i + m_i.$$

Substitution of d_i from the second of these equations into the first and solving for m_i gives imports as a linear function of domestic output:

$$m_i = a_i + b_i q_i$$

To estimate these regressions, we need data on domestic output, *out*; it is found in the OUT.DAT file.

(In real economies, it is quite normal to have data on output in years for which the complete input-output table is not available. You may, however, wonder how we made it up for *Tiny*. In Section 16.7, we will introduce a series of input-output coefficient tables for 1995 – 2003. From the data already given for PCE and imports, and by moving the 2000 final demand vectors for investment, government, and export by the corresponding historical totals, final demand vectors for these years were calculated; with the time-varying input-output table, the implied domestic outputs shown in the OUT.DAT file were calculated.)

The G7 command file to do the regressions is shown on the left below, and the resulting IMPORT.EQN file is shown on the right. The program to compute the imports from these equations is given in the box on the following page and may be found in MODEL4.CPP.

IMPORTS.REG – Estimate Imports

```
# Estimate the import equations
bank tiny
vam hist b
dvam b
add impR.dat
add out.dat
vc out = LINV*fd

# Regress imports on outputs
lim 1995 2003 2003
punch import.eqn 4 2 2003
ti Agricultural Imports
r im1 = out1
gr *
ipch im 1 L
ti Mining Imports
r im2 = out2
gr *
ipch im 2 L
ti Manufacturing Imports
r im4 = out4
gr *
ipch im 4 L
ti Service Imports
r im7 = out7
gr *
ipch im 7 L
punch off
```

IMPORT.EQN – Equation File for Imports for *Tiny*

```
 4 2 2003
im 1 L 2
 1 2
     0.253335      18.6513   0.00365866
im 2 L 2
 1 2
    -0.104578     0.740326     0.170614
im 4 L 2
 1 2
   -0.0630504     -67.3531     0.282823
im 7 L 2
 1 2
     0.235647     -14.1671     0.049055
```

We now need a version of Seidel that will compute imports simultaneously with outputs. It is shown in the box below and may be found at the end of the MODEL4.CPP file. The prototype for it is found near the top of the MODEL4.CPP file:

```
// Prototype for Seidel used in TINY
short Seidel(Matrix& A, Vector& q, Vector& f, IVector& triang,float toler);
```

so that when a reference is made to it later, the C++ compiler will know how to set up the call to it and how to handle the return. In this same file, in the *spin()* function, right after the PCE and investment functions have be calculated, we find the lines:

```
inv = invtot[t]*invc;
```

```
gov = govtot[t]*govc;
ex  = extot[t]*exc;
fd = pce + gov + inv + ex;
Seidel(AM, out, fd, dump, triang, toler);
imtot[t] = im.sum();
```

The first three compute final demand vectors as we have done up to now. Then the *fd* vector is calculated as the sum of these components. Note that imports have not been subtracted.

The Import Function from the MODEL.CPP File

```
// Importfunc() -- Import functions for TINY
int importfunc(Vector& q){
   int n, i,j,t1;
   float imp;
   char which;

   if(t < MacEqStartDate) return(0);
   n = impeqn.neq; // Number of equations
   // pdisinc is personal disposable income. It is a global macro variable.

   // Loop over the equations
   for(i = 1; i <= n; i++){
      j = impeqn.sec(i); // j is the sector number of this equation.
      which = impeqn.type(i);
      if(which == 'L'){
         imp = impeqn[i][1] + impeqn[i][2]*q[j];
         }
      else {
         printf("Unknown equation type %c in importfunc, sector %d.\n",
            which,i);
         tap(); //Pause so that error message can be read.
         continue;
         }
      // Apply rhoadjustment
      // Note the use of i and j in the following statement.
      im[j] = impeqn.rhoadj(imp,im[j],i);
      }
   im.fix(t);
   return(n);
   }
```

In the *Seidel()* function, a potentially infinite loop is started by the code of which the outline is:

```
iter = 0;
while(1){
    . . . .
    iter++;
    if(dismax < toler || fdismax < .000001) break;
    if(iter > 25 )return(ERR);
    }
return(OK);
```

The key to getting out of this loop successfully is clearly to get the *dismax* variable down to less than toler, where toler is the error tolerance for the Seidel process which was given in the call to *Seidel()* and *dismax* is the maximum discrepancy between the output of a sector calculated on the current

iteration of the Seidel process and the output of the same sector calculated in the previous iteration. The exit from the while loop when the iteration count passes 25 is just a safety net to avoid a hung computer if the process fails to converge.

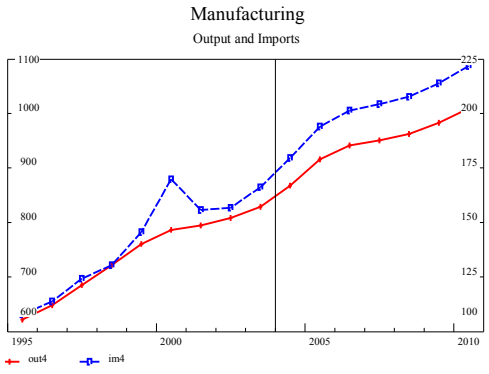
The first thing done under the while loop is to compute imports *im*, with a call to the import function. These imports are then subtracted from the given fd vector to get the net final demand vector, *f*, with which the rest of this iteration of the Seidel process takes as its final demand. Note that the calculation of the import vector may involve not only the import equations but also rho-adjustments and perhaps other fixes. Convergence on the *q* or output vector will imply convergence on the import vector as well.

The most of the rest of the code of the *Seidel()* function just implements the process as explained above and should be fairly transparent. There is, however, one non-standard wrinkle which needs explanation. It gives the process the capability of accepting a pre-determined output for one or more products. Petroleum extraction in the United States is a good example; demand beyond some capacity level must be imported or dealt with in some way as a shortage. If *i* is a normal, non-constrained product, then in column 18 of the SECTORS.TTL file its row should have the letter 'e'. The 'e' means that the product uses the equation to determine output. Here are a few lines from *Tiny*'s SECTORS.TTL:

```
Agricul     ;1   e   "Agriculture"
Mining      ;2   e   "Mining and quarrying"
Elect       ;3   e   "Electricity and gas"
```

Whatever letter is in that column will become the value of *outfix(i)*. When our routine finds a value of 'e' in *outfix(i)*, it will set the output of product *i* equal to the demand which has been calculated. If, however, *i* is a constrained product, the routine keeps the initial, presumably exogenously set, value of *q[i]* and puts the difference between it and calculated demand into the vector called *dump*. What should be done with *dump* is then up to the model builder who is using this feature. Since *Tiny* is not using it, we will do nothing with *dump*.

At this point you may want to refresh your memory of the Seidel process, but after doing so, all the coding in the *Seidel()* function shown in the box on the next page should make sense. When it does, build and run the new model and make graphs of imports and outputs. (You will need to copy MODEL4.CPP to MODEL.CPP .)



The above graph shows the similarity that we now find in the forecasted course of outputs and

imports for Manufacturing. The scale for the two curves, however, is different; the scale for output is shown on the left and that of imports on the right. The imports – the blue, upper line marked with squares – are actually much less than output, but the pattern of movement is similar. (The right vertical axis for graphs drawn with the *G7* "mgr" command may be set by the "hrange" command.)

```
/* Seidel() Version 2, with imports .************************************

short Seidel(Matrix& A, Vector& q, Vector& fd, Vector& dump, IVector& triang,
    float toler){
    short i,j,k,first,last,n,iter,im1,imax;
    float discrep,dismax,fdismax;
    double sum;
    Vector f(fd.rows()); //scratch vector

    iter = 0; // Iteration count
    n = A.rows();
    first = A.firstcolumn();
    last =  A.lastcolumn();

    if(q.rows() < A.rows()){
        printf("In Seidel, the solution vector is not large enough.\n");
        return(ERR);
        }
    dump.set(0.);

    while(1){
        // Compute imports
        importfunc(q);
        f = fd - im;
        dismax = 0;
        for(k = triang.First(); k < triang.First()+ triang.numelm(); k++){
            i = triang[k];
            sum = f[i];
            for(j = first; j <= last; j++)
                sum += A[i][j]*q[j];
            sum -= A[i][i]*q[i]; // Take off the diagonal element of sum
            sum = sum/(1.- A[i][i]);
            if(outfix[i] != 'e'){ // Override the equation
                // Dump error into dump
                dump[i] = q[i] - sum;
                sum = q[i];
                }
            else dump[i] = 0;
            discrep = fabs(sum - q[i]);
            if(discrep > dismax){
                dismax = discrep;
                imax = i;
                if(fabs(sum)>1.0e-3) {
                    fdismax = fabs(sum);
                    fdismax = dismax/fdismax;
                    }
                }
            q[i] = sum;
            arith("in Seidel",i);
            }
        iter++;
        if(dismax < toler || fdismax < .000001) break;
        if(iter > 25 ){
            printf("No convergence in %d iterations. Discrep = %7.2f in sector %d.\n",
            iter, dismax, imax);
            return(ERR);
            }
        }
    printf("Seidel iterations: %d ",iter);
    return(OK);
    }
```

16.6. Speeding Up Solutions with Read and Write Flags

So far, we have been reading and writing the value of every vector and every matrix from the VAM file in every period. That is neither desirable nor necessary. Take the AM matrix for example. We know perfectly well that at present we do nothing to change it in the program, so writing it back to the VAM file at the end of each year's calculation is a waste of time. In the case of the *out* vector, we certainly want to write the value after it has been computed, but in the forecast period the value of *out* in the VAM file on the first run of the model will be all zero. That is not a good starting value for the Seidel process. It would be much better just to start with the previous period's value. Thus, we would want to read it only in the initial year.

Such considerations have lead to the introduction of read and write flags for the vectors and matrices which are set when they are resized in MODEL.CPP. For example, so far we have had as the first line of the resizing commands

```
out.r("out");pce.r("pce");gov.r("gov");
```

which is equivalent to

```
out.r("out",'y','y');pce.r("pce",'y','y');gov.r("gov",'y','y');
```

The second argument – the first 'y' – in one of these function calls is the read flag; the third, the write flag. Possible values for the read flag are:

- 'y' yes, always read the vector or matrix
- 'n' no, never read the vector or matrix
- 'i' read the vector or matrix in the initial year of the run
- 'a' read the vector or matrix if there is data available for it.

Possible values of the write flag are:

- 'y' yes, always write the vector or matrix
- 'n' no, never write the vector or matrix

The default value of both flags is 'y'; that is to say, if no flags are specified, both are assumed to be 'y'. If only one is specified, it is interpreted as the read flag. To specify a write flag, we must also specify the read flag, even if it is 'y'. For example, to set the read flag for the AM matrix to 'y' and the write flag to 'n', we would use the call

```
AM.r("am", 'y','n');
```

To set the read flag for the *out* vector to 'i' but the write flag to 'y' , it is enough to use the call

```
out.r("out", 'i');
```

The read flag 'a' can be quite convenient to read the values of a vector in years when it is known but not in later years. Its use, however, requires telling the program the last year of data for the vector or matrix in question. To do so, the first step is to remove the comment indicator (//) in front of the line

```
// lastdata(); // Use of lastdata function requires that the LastData file exist.
```

which is found just above the *resize()* commands for vectors. The call to the function *lastdata()* will read a file which must be called LASTDATA. Here is a possible LASTDATA file for *Tiny*:

```
pce 2003
im 2003
out 2000
```

It simply lists one or more of the vectors in the VAM file and the last year for which data is available for each.

Because *Tiny* is so tiny, you will not notice any acceleration of the solution by the use of these flags. For big models, however, they can make a small but measurable difference in solution time. You should experiment with them in *Tiny*, but since there is not much in the way of results to show, I will not give a more explicit application.

16.7. Changing Input-Output Coefficients and Prices

Up until now, our input-output coefficient matrix has remained constant, as have the coefficients for the value-added components: depreciation, labor income, capital income, and indirect taxes. Consequently, there has been no need to consider changes in relative prices. We must not leave *Tiny* without introducing coefficient change and relative prices, for input-output analysis has often been unjustly reproached for assuming constant coefficients. In fact, it is precisely input-output analysis which enables the specification of changing input-output coefficients. In this section, we will present Model 5, which incorporates prices and coefficient change.

I prevailed upon the *Tiny* statistical bureau to construct a coefficient matrix for 1995 and drew upon the collective wisdom of a group industry specialists to project the table to 2010. The results of these labors, together with the matrix for 2000 with which we have been working, are contained in the files AM1995.DAT, AM2000.DAT, and AM2010.DAT. For intervening years, we will have to rely upon linear interpolation. The same groups provided us with past history and projections of the labor income coefficients for the same two years. They are in the file LABC.DAT.

We need to significantly expand the VAM file so we will start from a new VAM configuration file, VAMP.CFG, which you will find on the disk. (The P in the filename stands for Prices.) We have added the vector of prices, *prices*, final demands in nominal prices, *fdN*, *pceN*, *govN*, *invN*, *exN*, and *imN*, the interindustry flow matrix in nominal terms, *FMN*, and two vectors, *fix* and *cta*, whose functions will be explained in the next section on fixes.

Corresponding to this new configuration is a new *G7* command file, TINYP.PRE, to create the initial VAM file for the model. In some ways it is simpler than before; we do not need to compute the AM matrix but

The TINYP.PRE File

```
# File to create VAM file for TINYP, TINY with Prices
vamcreate vamp.cfg hist
vam hist b
dvam b
# Bring in data on outputs and final demands
add out.dat
add pceR.dat
add impR.dat
# Bring in the intermediate flow matrix
add flows.dat
# Bring in the final demand vectors
add fd.dat
# Bring in the value added vectors
add va.dat
fdates 2000 2000
# The following are element-by-element vector divisions
vc depc = dep/out
vc capc = cap/out
vc indc = ind/out
# Compute share vectors of final demands
vc invc = inv/invtot{2000}
vc govc = gov/govtot{2000}
vc exc  = ex/extot{2000}
# Read I-O coefficient matrices
add AM1995.dat
add AM2000.dat
add AM2010.dat
# Interpolate matices for missing years
fdates 1995 2010
lint AM
show AM r 4
# Keep value added coefficients, except labc, constant
dfreq 1
f one = 1.
index 2000 one depc
index 2000 one capc
index 2000 one indc
#Read labc for 1995, 2000, and 2010
add labc.dat
lint labc
# Keep the structure of some final demand columns constant
index 2000 one invc
index 2000 one govc
index 2000 one exc
# Compute final demand vectors for historical years
fdates 1995 2003
vc inv = invtot*invc
vc gov = govtot*govc
vc ex  = extot*exc
store
```

rather just read it for 1995, 2000, and 2010. The one new command following the reading of these matrices is simply

```
lint AM
```

where "lint" stands for "linearly interpolate." This command will linearly interpolate[5] the values of each and every cell between the years where non-zero values are given. The same command is applied a few lines further down to the *labc* vector. The lint command works over the range specified by the last previous "fdates" command. When you have run this file through *G7*, you will have created a new HIST.VAM file. Copy it to BASE.VAM with the following *G7* command:

```
dos copy hist.* base.*
```

The MODEL.CPP file for the model with prices is on the disk as MODEL5.CPP; copy it to MODEL.CPP. Near the top of the file, resizing functions for the new vectors and matrix have been added. The new elements in the heart of the *spin()* function are shown in bold in the box below.

Excerpt from MODEL5.CPP File

```
for (t = godate; t<= stopdate; t++) {
   . . . .
   // Start of code particular to TINY:

   // Compute total value-added coefficient vector.
   vac = depc+labc+capc+indc;
   // Compute prices
   PSeidel(AM, prices, vac, triang, 0.000001);
   // The loop for convergence on pcetot and invtot.
   while(Iteration < 20){
      . . . .
      }
   // Set error for rhoadjustment
   setrho = 'y';
   if(t >= pceeqn.rhostart)  pcefunc();
   if(t >= impeqn.rhostart)  importfunc(out);
   setrho = 'n';
   // Here when both Investment and PCE have converged
   // Compute flow matrix in current prices
   FM = AM % out;
   FMN = prices % FM;
   // Put output and final demands into nominal prices
   outN = ebemul(prices,out);
   fdN = ebemul(prices,fd);
   pceN = ebemul(prices,pce);
   invN = ebemul(prices,inv);
   exN  = ebemul(prices,ex);
   imN  = ebemul(prices,im);
   govN = ebemul(prices,gov);
   // End of code particular to TINY
   . . . .
   }
```

5 If this term is unfamiliar, it means to fill in missing data by drawing a straight line between two adjacent known points.

The first addition is to compute the total value added coefficient vector and then compute the *prices* vector by a call to *PSeidel()*. This routine, which may be found in SEIDEL.CPP, solves the equations

$$p = p\,A + v$$

where p and v are row vectors. If the sectors of an input-output matrix have been arranged in an almost triangular order for solution of the output equations by the Seidel method, the *opposite* order will most likely be good for solving for prices. Hence, *PSeidel()* uses the same *triang* vector as does *Seidel()* but takes the sectors starting from the end of the list and working towards the beginning. The remaining new commands are just to calculate the values of the nominal vectors and of the nominal flow matrix, *FMN*. This last calculation uses Interdyme's % operator. It is used between a vector and a matrix. If the vector is on the **right**, it multiplies each **column** of the matrix by the corresponding element of the vector. If the vector is on the **left**, it multiplies each **row** of the matrix by the corresponding element of the vector.

A final note on the economic interpretation of the prices we have calculated may be helpful. They are, in fact, relative prices and the *numeraire*[6] with price equal to 1.0 is the quantity of labor which could be bought for one dollar in the year 2000.

To obtain truly nominal prices, we need to model inflation, and to model inflation, as we already know from the *Quest* model in Part 2, we need to model employment and unemployment. Further, we need to model savings behavior more carefully and consider the role of money and interest rates in both savings and investment. All of these things can be modeled using the techniques we already have, so they make a better exercise for you than subject for me.

We can introduce optimizing in Interdyme models in exactly the same way as we did in *Quest*. All we need do is to specify the objective function and the regression coefficients to be varied in exactly the same way as in a macro model and then check the "Optimizing" radio button on the panel of window which comes up when we click Model | Run dyme. At present, optimization works only with respect to the coefficients of macro equations, not for the coefficients of systems of detached-coefficient equations. There are no problems of principle involved in extending the code to handle coefficients of those equations also, just writing some housekeeping code. Finally, although there is a "Stochastic simulation" radio button on that same panel, it does not work at present. Much of the necessary code has been brought over to Interdyme, but somewhat more work is needed to activate it.

It is tempting to go on elaborating the fictitious *Tiny* economy with all these features, but you surely have a real model you are eager to get to work on, so we will give *Tiny* no further features, for there are no more major new techniques that need to be explained. We do, however, need to deal with how to use fixes to discipline *Tiny* or any model if it misbehaves.

Exercise

1. Make up employment and labor force data for *Tiny*, and estimate and put into the model employment equations. Have the model compute unemployment. Modify the savings equation so that the model provides employment for roughly 96 percent of the labor force, though the exact level may vary cyclically.

6 Unit of value on which other values are based.

16.8. Fixes in Interdyme

For smaller models, the use of the .XG files described in section 16.1 in the discussion of *IdBuild* is a quick and flexible way to input assumptions about the exogenous macro variables in the model. Analogous to the use of .XG files, one can use *G7* to write out the values of exogenous vectors and matrices in the VAM file for the forecast period. However, as the model grows in size and complexity, the need arises for more sophisticated ways to make assumptions about variables, both endogenous and exogenous variables. The use of "fixes" has evolved to satisfy this need. Fixes are assumptions about variables which are stored in a file, and read using a fixer program. A given model (including historical data, equations and code) and a given set of fixes will determine a scenario or simulation, in a way that will give reliable, repeatable results. Base and alternate scenarios can be developed by suitable modifications of the fixes files, with both sets of fixes run with the same model.

Fixes, as used here, are ways to make a model work the way we want it to, not necessarily the way that emerges from its equations. The power that fixes give over a model can certainly be abused. Nonetheless, they are extremely useful and powerful. Suppose, for example, we wish to consider the impacts of some event which the model equations were never confronted with, like a natural disaster or a massive overhaul of the health care system. Then a fix is the natural way to convey to the model that it needs to work in a way not described by its equations. Another use of fixes is to force the detailed data to agree with published aggregate data. The aggregate data is usually available up to a current quarter or month, whereas sectoral data is only published with a lag, sometimes of several years. Control totals can be applied to the vector elements, and the projection made using the individual equations may be scaled. This is an example of a "group fix", where a defined group of elements of a vector may be fixed as a unit.

Interdyme has three types of fixes, those for macro variables, those for vectors and matrices, and a special type which we have already seen for industry outputs. This section will primarily serve as a useful reference to the fixes files and the commands that can be used within them. We will start with describing the fixes for macro variables.

Macro Variable Fixes

Macro variable fixes are fixes applied to variables of type Tseries, which are defined using the *Idbuild* program described above. These fixes work much like those of models built with the G-Build combination, but also have much in common with the vector fixes described in the next section. The program that handles the macro variable fixes is called *MacFixer*. The input to *MacFixer* is a file prepared by the user with a text editor. It generally has the extension .mfx . Once this file has been created, the fixes can be processed by running the program *MacFixer* The results of this program are written to a "macro fix bank", which is essentially a G bank, which can be read with *G7*. The root name (the part of the filename before the dot) of the macro fix G bank is passed to *MacFixer* through a file named MACFIXER.CFG. It must also be passed to the simulation program by the form that opens on choosing the command Model | Run Dyme .

The configuration file MACFIXER.CFG contains the name of the text input file, the root name of the G bank file used for base values for the index and growth-rate fixes (this would normally be the G bank created for use with the simulation program, such as HIST.BNK), the name of the G bank which will contain the values of the fixes, and the name of the output check file. This last file shows

the values of each fix in each year, and serves as a check on the results in the binary file.

While it is up to the user to name files, it makes sense to give files for the same simulation the same root name. A simulation that involves low defense expenditures, for example, could have a G bank file called LOWDEF.BNK, and a *.mfx* file called LOWDEF.MFX.

Several varieties of macrofixes that may be given. They can generally be divided into *absolute* or "hard" fixes, and *modifier* or "soft" fixes. The syntax for the hard fixes ("ovr", "ind" and "gro") is given first.

ovr *overrides* the result of the equation with the value of the time series given. Values between given years are linearly interpolated. In the example below, the macro fix program would calculate and override a fix series that starts in 1992, ends in 2000, and moves in a straight line between the two points. For example

```
ovr uincome$
 92  154.1
 2000  182.3;
```

would override the value of the forecast of *uincome$* with the values shown for the years shown. Note that year can be either 2-digits or 4-digits (they are all converted to 4-digits in the program). If *uincome$* were an exogenous variable, then it *must* receive a fix. If it were endogenous, the above fix would replace the values forecasted by the equation.

ind is a variety of the override fix that specifies the time series as an index. There must be data in the VAM file for the item being fixed up until at least the first year of the index series specified. The value for the item in that year is then moved by the index of the time series given by the fix lines. For example,

```
ind invtot
 2003  1.0  1.03  1.08  1.12  1.15
 2008  1.21 1.29  1.31
```

will move the value of *invtot* in 2003 forward by the index of the series given, and will replace the calculated value of *invtot* by this value when the model is run.

gro is a type of override fix that specifies the time series by growth rates. For the growth rate fix to be legal, there must be data in the VAM file up until at least the year before the first year of the growth rate fix. Missing values of the growth rates are linearly interpolated.

```
gro wag01
 1993  3.1
 2000  3.4;
```

stp is a type of override fix that specifies the growth rate in a series of steps. It is like "gro" except that a growth rate continues until a new one is provided. A value for the final period is necessary.

```
stp wag01
 1993 4.1
 1995 4.5
 2000 5.0;
```

The next two types of fixes ("cta" and "mul") are the *modifier* or "soft" fixes

cta does a constant term adjustment. That is, it adds or subtracts the value of the time series to the result of an equation or a previous fix. The time series is provided by the fix definition. For example,

```
cta nonagincome
  1992 .0001
  1995  200
  2000  180;
```

is a constant term adjustment for nonagricultural income from 1992 to 2000. Intermediate values are of course linearly interpolated.

mul multiplies the equation's forecast by a factor specified by the data series on the following line. For example:

```
mul ulfi$
 1992  1.0
 1995  1.05
 2000  1.10;
```

multiplies the forecast results for the macrovariable *ulfi$* by the factors shown. Values of the multiplicative fix between the years shown are linearly interpolated, as usual.

Next is the "rho" fix, which plays a very important role for macrovariables.

rho is the *rho-adjustment* fix, familiar from macromodels. It may, in fact, be considered a fully legitimate part of the model. The format is:

rho <depvar> <rho_value> [rho_set_date]

where

depvar is the name of the dependent variable
rho_value is the value of rho.
rho_set_date is the year in which the rho-adjustment error is to be calculated. If none is provided, it is set in the year specified by the Macro Equation Start Date specified on the form when starting a run.

for example:

```
rho invtot .40 2003
```

tells the model to apply a rho-adjustment to the variable *invtot* using the value .40 for rho, and starting the rho-adjustment in 2003.

A rho fix with a rho_set_date works like a "skip" (see below) in years before the rho_set_date. Unlike rho fixes in macromodels, an endogenous variable can have a "rho" fix in conjunction with and a "cta" or "mul" fix. The rho-adjustment is applied before the other fix.

skip is the simplest type of fix. It simply skips the equation and uses the values in the model G bank. For example:

```
skip invn$35
```

would skip the equation for the macro variable *invn$35*, and use the value already in the model G bank.

The next several fixes ("dind", "dgro" and "dstp") are the family of *dynamic* fixes. Unlike the "ind", "gro" and "stp" fixes which establish the fix values before the model is run, and write these values to the fixes file, dynamic fixes usually begin in some future year, and base the index or growth rate on the value of the variable in that year, as it has been calculated by the model. In other words, the same fix will probably yield different results in different model runs.

dind is the "dynamic index fix". This fix can start in any year, and does not rely on historical data being present in the databank. Rather, the fix is calculated during model solution when the first year used is specifying the fix is reached. For example, if the fix is

```
ind invtot
 2005  1.0  1.03  1.08  1.12  1.15 1.20
```

and a forecast is begun in 2003, then the equation will be used for 2004 and 2005, but the value of *invtot* in 2006 will be 103 percent of whatever value was calculated for 2005.

dgro is the dynamic version of the growth rate ("gro") fix. This fix can start in any year, and does not require data to be available in the databank for the starting year of the fix. The growth rate is always applied to the value of the variable in the previous period.

dstp is the dynamic version of the "stp" fix.

The final three types of fixes ("eqn, "fol", and "shr") are very powerful. They enable the model user to play some of the roles of the model builder. Using an equation fix, a replacement equation for a variable can be specified in the fixes file, which can use any legal G7 expression of other macrovariables, vector variables and matrix variables. The body of the fix plays a special role in the fix, as shown below. Follow ("fol") and share ("shr") fixes are really special variants of the more general equation fix, but the need for them is so common that they have been made available as special types. We will now describe these fixes for macrovariables.

eqn is the *equation* fix. This type of fix lets you dynamically introduce a new equation relationship into the model at run time. The advantage of this type of fix is that users of the model who are not programmers can introduce their own assumed relationships into the model, without having to change the model program code. It is also helpful for prototyping a model, where you want to quickly try out different equation relationships to see how they work, before coding them into the model.

The equation fixes use the same expression syntax as used in the "f" command and other commands in *G7*. The format for equation fixes for macrovariables is:

```
eqn <Macroname> = <expression>
        <year> <value> [<value> <value> ...]
        <year> <value> [<value> <value> ...]
```

where: <Macroname> is a legitimate name of a macrovariable, <expression> is a legitimate

expression, as described below, and the <year> <value> entries are in the same format as the data for other fixes, but indicate the years for which the equation fix is to take effect. They also represent the time series for a special variable called *fixval*, which can be used within the equation expression. This *fixval* variable can be used wherever a vector or macrovariable could be used.

Just about any expression that is legal in *G7* is legal for an equation fix, except that only a subset of functions are implemented. These functions are: @cum, @peak, @log, @exp, @sq, @sqrt, @pow, @fabs, @sin, @pct, @pos, @ifpos, @pct, @rand and @round.

Lagged values of any order can be used, with the constraint that they must not be before the starting year of the model G bank (DYME.BNK). Macrovariables are read directly from memory. Lagged values of vector variables are read from the VAM file. Therefore, you can use a lagged value of any vector as far back as the starting date of the VAM file, and you are not limited by whether or not that vector has been declared to store lagged values in memory in VAM.CFG.

Examples:

```
# Make the T bill rate equal to the average inflation plus some percent,
#   specified in "fixval".
eqn rtb = .34*gdpinf + .33*gdpinf[1] + .33*gdpinf[2] + fixval
  2016 1.0
  2030 1.5;
```

Though it is not necessary to know in order to use the function, you may be curious as to where the equation is stored and how it is calculated. To continue with the example just given, the *Macfixer* program stores a fix by the name of *rtb:e* and the values of *fixval* as the value of the fix. Then at the end of index file for this bank (MACFIXES.IND if MACFIXES is the name of the bank), where it does not interfere with *G7*'s use of the bank, it labels and stores the text of the equation. When the fix is to be applied, this text is read and the value calculated with code borrowed from *G7* for calculating *f* commands.

fol is the *follow* fix. The follow fix allows you to specify that a macrovariable should move like some other quantity, which may be specified as a general expression involving vector and macrovariables, just like the equation fix.

The general format for the follow fix is:

fol <Macroname> = <expression>
<year> <value> [<value> <value> ...]
<year> <value> [<value> <value> ...]

The variable "fixval" should not be used in the follow fix expression. Its purpose is to specify a growth rate to add to the growth of the expression.

For example, if we would like to specify that Medicaid transfer payments grow like real disposable income per capita, plus 0.1 per cent, we could write:

```
fol trhpmi = di09/pt
2016 0.1
2040 0.1
```

shr is the *share* fix. This fixed is used to specify that the macrovariable should be a certain share of another variable or expression, with the share specified by the fix value. Actually, the "share" is just a multiplier, so it can be any number.

The general format of the share fix is:

shr <Macroname> = <expression>
<year> <value> [<value> <value> ...]
<year> <value> [<value> <value> ...]

In the share fix, the fix value is the multiplier or share to multiply by the right hand side expression.

When the input file specifying the fixes as described above is ready, it should be saved with ".mfx" as the extension of its file name.

On the *G7* main menu, then click Model | Run Dyme Model, and the fixer programs (*MacFixer* and *Fixer*) will be run before running the model. As described previously, the macro fixes are written into a file which is structured like a G bank. For example, in the MACFIXER.CFG shown below for *Tiny*, the name of the output fixes bank is "MacFixes". Therefore, *MacFixer* will create the files MACFIXES.BNK and MACFIXES.IND, which may be assigned and viewed in *G7* just like any other G bank.

MACFIXER.CHK

```
Input fix file    ;Macfixes.mfx
Output fixes bank ;MacFixes
Model G bank      ;tmp
Output check file ;MacFixer.chk
```

The names of the series in this G bank are formed by concatenating the series name with a colon followed by a single letter indicating the type of fix to be applied to that variable. Thus, the fix

```
cta invtot
2004 10.
2010 17.
```

will put into the MACFIXES data bank a variable by the name *invtot:c* whose values will be linearly interpolated between 2004 and 2010. If you create a file by the name of MACFIXES.MFX with exactly this fix in it, save the file and click Model | Run Dyme Model to create the MACFIXES G data bank, you can then open up the bank, and give G7 the commands:

```
bank macfixes b
lis  b
```

and see that the bank has a variable by the name of *invtot:c*. You can then do

```
type  b.invtot:c
```

and see the interpolated add factor. Alternatively, you can look in the MACFIXES.CHK file and see

```
Fix Number 1: cta fix on invtot
  2003       10.00       11.00       12.00       13.00       14.00
  2008       15.00       16.00       17.00
```

which should assure you that the interpolation has been done correctly. This file also serves as a record of the assumptions made for a particular run.

The correspondence between fix types and codes suffixed to the variable names are: skip ('k'), ovr ('o'), cta ('c'), ind ('i'), gro ('g'), stp ('s'), rho ('r'), and eqn (‘e’).

Macro fixes provide an alternative way to supply values of exogenous variables. Exogenous variables may be put into the "hist" bank in the process of running *Idbuild*. If the variable appears in no *.sav* file for a macro equation, then it is included in the PSEUDO.SAV file. The standard way of providing the values of the exogenous variables is then through "update" or other commands in *G7*. Another possibility for providing exogenous values is to have a special run of *G7* with the "hist" or other bank as the workspace bank. Finally, one can provide the exogenous values as macrofixes. For example, if we want *disinc* to be an exogenous variable, then -- however we are going to provide the values -- we need the statement

```
f disinc = disinc
```

in the PSEUDO.SAV file. To use the macrofix method of assigning values, we need in the code of the model the statements

```
depend=disinc[t];
disinc[t] = disinc.modify(depend);
```

We could then provide the values with "ovr", "ind", "gro", or "stp" commands to the macrofix program, for example, by

```
gro disinc
 1995 3.0
 2000 3.5
 2005 4.0;
```

This method has the advantage of keeping all the fixes which constitute a scenario in one place. It also allows the use of the "gro" and "stp" fixes, which may be convenient. It has the disadvantage of adding an additional series to the banks which constitute the model and an additional statement within the model.

Vector and Matrix Fixes

The vector fixes are more complicated than macro fixes because they can apply to individual elements of a vector, to the sum of a group of elements, or to the sum of all elements in the vector. However, the format of the vector fixes is similar to that of the macro variable fixes, described above. Matrix fixes at the current version are still rather simple, one fix being applicable to only one cell of a matrix. The preparation of the vector and matrix fixes is the work of the *Fixer* program.

Unlike the macro fixes, which are automatically applied when a macro regression equation is calculated, vector and matrix fixes are applied where the model builder specifies. At the point where the fixes for the vector *x* should be applied, the model builder must put into the program the line

```
x.fix(t);
```

The input to *Fixer* is a file prepared by the user in a text editor. It should have the extension *.vfx*. *Fixer* also reads the definitions of static groups of sectors and writes them into the GROUPS.BIN file which can be used both by the simulation program and by *G7*. To use the *Fixer* program, it is essential that the model's VAM.CFG file should have a vector called "fix" with enough rows to allow one for each fix. As Fixer reads the fixes from the input file, it stores the numerical values of the fixes into this "fix" vector in the vam file. It also creates a "fix index" file, which will have the extension *.fin* and tells the simulation what to do with each fix. Finally, it produces a binary file with the definitions of groups, called GROUPS.BIN. If *G7* has already produced a GROUPS.BIN file, *Fixer* reads it and may add to it.

For vectors, fixes may apply to a single element or a group of elements. The concept of a “group” is central to the working of *Fixer*. Basically, a group is simply a set of integers, usually representing sectors in the model. Defining groups is useful because we often want to impose a fix on a group of elements in a vector. For example, we may want to control the total exports of the chemical manufacturing sectors. We might then create a group named “chem” which would contain the sector numbers of all the sectors in question. The command for defining a group for *Fixer* is “grp <groupname>”, where the groupname can be a number or a name. The sectors defining the group are then entered on the next line. For example:

```
grp 1
  7 10 12
```

creates a group called “1” consisting of the sectors 7, 109 and 12. The '-' sign means consecutive inclusion. Thus:

```
group zwanzig
 1-20
```

consists of the first twenty integers. Parentheses mean exclusion. Thus:

```
group duo
  :zwanzig (2 - 19)
```

makes the group "duo" consist of the integers 1 and 20.

When a group is referenced after it is defined, its name must be preceded by a colon, as shown when "zwanzig" was used in the definition of "duo" above. Names of groups are case sensitive; commands for Fixer must be lower case. Groups can be defined anywhere in the input file before the first time you used them. If you try to redefine an existing group, the program will complain, unless the new group has the same or less elements than did the old group. References to other groups can be used in new group definitions only if the groups referenced have already been defined.

Interdyme provides a number of ways for a fix to work. In all of them, a time series is specified by the fix. The forms of the fix differ in how they obtain and in how they apply this time series. The basic format of the input file for a vector fix is:

<command> <vectorname> <GroupOrSector>

followed on the next line by a year and some values of the fix. The basic format of the input file for a matrix fix is:

<command> <matrixname> <row> <col>

The fixes available for vectors and matrices are for the most part the same as those for macrovariables, with the exception that you need to provide the <GroupOrSector> for a vector fix, and the <row> and <col> for a matrix fix.

ovr overrides the result of the equations with the value of the time series given. Again, intermediate values are linearly interpolated. In the example below, the fix program would calculate and override fix series that starts in 1992, ends in 2000, and moves in a straight line between the two points. For example,

```
ovr ex 10
  1992  154.1
  2000  182.3;
```

would override the value of the forecast of element 10 of the "ex" vector (probably exports) with the values shown for the years shown. As an example of a matrix fix,

```
ovr am 1 9
  1990 .23
  1995 .26
  2000 .28;
```

would override the value of the A-matrix in the Vam file for element (1,9), from 1990 to 2000. As before, missing values are linearly interpolated.

ind, dind

is a variety of the override fix that specifies the time series as an index. There must be data in the vam file for the item being fixed up until at least the first year of the index series specified. The value for the item in that year is then moved by the index of the time series

given by the fix lines. For example,

```
ind pceio :zwanzig
 1982  1.0  1.03  1.08  1.12  1.15
 1997  1.21  1.29  1.31  1.34;
```

will calculate the sum of the elements of the pceio vector included in the group "zwanzig" in 1982, will move that sum forward by the index of the series given, and will impose that control total on the those elements when the model is run.

The “dind” version of the fix can start in any year, and indexes the series to the value of the expression in the starting year of the fix.

gro, dgro

is a type of override fix that specifies the time series by growth rates. For the growth rate fix to be legal, there must be data in the vam file up until at least the year before the first year of the growth rate fix. Missing values of the growth rates are linearly interpolated.

```
gro out 10
 1983  3.1
 2000  3.4;
```

The “dgro” version of the growth rate fix can start in any year, and always calculates the series in the present period based on the value in the previous period.

stp, dstp

is a step-growth fix. It is like “gro” except that a growth rate continues until a new one is provided. A value for the final period is necessary.

```
stp out 1
 83  4.1
 95  4.5
 2000 5.0;
```

The “dstp” version is the dynamic version, which can start in any year. It is just like “dgro”, except for the method of interpolation of the fix values.

mul multiplies the equation forecast by a factor specified by the data series on the following line. For example,

```
mul im 44
 1992  1.0
 1995  1.05
 2000  1.10;
```

multiplies the forecast results for imports of sector 44 by the factors shown. Values of the multiplicative fix on imports between the years shown are linearly interpolated.

cta does a constant term adjustment. That is, it adds or subtracts the value of the time series to the result of the equations. The time series is provided by the fix definition. For example,

```
cta def :Alice
  1992 .0001
  1995  200
  2000  180;
```

is a constant term adjustment for defense expenditures of all sectors in the Alice group.

Intermediate values are linearly interpolated.

eqn The equation fix for vectors works in the same way as the version for macrovariables, with the exception that the name of the vector must be separated from the sector number by a space. For example:

```
# Make the pce deflator for category 3 grow like the aggregate PCE deflator,
#   based on the ratio in 1997, from 1998 to 2010.
eqn cprices 3 = cprices3{1997}/apc{1997} * cprices3
   1998 1
   2010 1
# Make corporate profits in sector 1 remain a constant share of total
corporate
#   profits, equal to the share in 1997:
eqn cpr 1 = cpr1{1997}/vcpr{1997} * vcpr
  1998 1
  2010 1
```

fol The follow fix specifies that an element or group of a certain vector should follow the expression on the right, plus or minus a certain growth rate, which can be specified in the body of the fix. It is often used to make imports of a certain commodity grow like domestic demand. For example, the following follow fix makes crude petroleum imports grow like domestic demand, plus 0.2 percent per year:

```
fol im 4 = dd4
1998 0.2
2030 0.2
```

shr The share fix takes the value of the body of the fix (fixval), and multiplies the right hand expression by it, before assigning the value to the left hand side variable or group. Like the follow fix, a typical use for this fix might be in controlling the relation between imports and domestic demand. The example below specifies the share of domestic demand for imports of Radio, television and video equipment:

```
shr im 42 = dd42
1998 .9
2000 .92
2030 1.0
```

When the input file as described above is ready and the FIXER.CFG file calls for its use, type "fixer" at the DOS prompt to invoke the *Fixer* program. *Fixer* is also run when you choose the menu option Model | Run Dyme Model in *G7*.

Output Fixes

The output fixes allow the values of output specified in the VAM file to override values computed by the input-output equations. There is then the question of what to do with the difference. Interdyme offers two possibilities: add any excess demand to imports or simply ignore the difference. The options are specified in column 18 of the SECTORS.TTL file, which

is where the names of the input-output sectors are. The options for this column are:

- e use the equation
- i add the difference to imports
- d put the difference in a vam vector named "dump", where nothing is done with it, but it can be displayed.

Answers

3.2 The new outputs are
166.14 55.21 222.30 763.57 426.48 206.41 812.58 148.00
and the primary resources required to produce them are
317.24 1351.84 268.34 226.58.

3.3 The net export of depreciation is -5.73.

3.4 The new price vector is (.92 .96 .89 .90 .71 .93 .96 1.00).

3.5 Net export of greenhouse gas production is -31.87.

CHAPTER 17. MATRIX BALANCING AND UPDATING

17.1. The RAS Algorithm

Making an input-output matrix from scratch for a country is a major undertaking often involving a group of ten or more people for a number of years. By the time the project is finished, the matrix refers to a year that is apt to seem part of ancient history. Hence the question arises: "Given an input-output table for a base year, is there a way to update it to a more recent year with less work than making the table from scratch?" In this updating, one usually has some data for the more recent year. One wants the matrix for this year, which we may call the target year, to conform to all those data.

Usually those data would include industry outputs, major GDP components, and value-added by industry. The value-added by each industry can then be subtracted from its output to give the total intermediate inputs by each industry. Thus, we would know the row total for each industry and the column total for each final demand column and for the intermediate use of each industry. An obvious check on the accuracy of this information is that the sum of the row totals equals the sum of the column totals. We will assume that this condition has been met, although meeting it is not always easy except by a rough scaling. Thus, we have the margins or frame for the table for the target year.

An initial guess of the inside of the table for the target year can then be made by assuming constant coefficients for the input-output coefficients and for the shares in each of the final demand vectors. More sophisticated initial estimates could also be made. One could use, for example, consumption functions to "forecast" the purchases of households. However the initial inside elements of the table are estimated, it is almost certain that they will not have the right row and column sums. Adjusting them to make them conform to these control totals is generally done by what has come to be called the RAS procedure, a name derived from notation in Richard Stone's description of the method in *A Computable Model of Economic Growth* (Chapman and Hall, London, 1962)[7].

The method is extremely simple in practice. First scale all of the rows so that each has the correct total. Then scale all the columns so that each has the correct total. The row sums are then probably no longer correct, so scale them again, and then scale the columns again, and so on until the scaling factors have converged to 1.0. The matrix at that point has the desired row and column sums. If A^t denotes the flow matrix at stage t of the operation, R^t denotes the row scaling factors at step t arrayed as the diagonal elements of an otherwise zero matrix, and S^t denotes the column scaling factors similarly arrayed, then the flow matrix at the beginning of stage t+1 is

$$A^{t+1} = R^t A^t S^t \tag{17.1.1}$$

The expression on the right gave rise to the name RAS, which may be pronounced as the three letters, or simply as "ras".

7 The idea had been mentioned by Leontief in the 1941 edition of *Structure of the American Economy*, but the idea seemed to pass unnoticed until applied by Stone.

17.2. Convergence of the Algorithm

The practice is simple, but will the process converge? To answer that question, we will need some notation. Let the original matrix be ***A***, whose elements we will denote by $a_{ij,}$ let b be the positive vector of required row sums and c be the positive vector of required column sums. The first condition is that ***A*** be non-negative. The second is simply that there must exist at least one matrix with zeroes where ***A*** has zeroes and positive numbers where ***A*** has positive numbers. This matrix must also have row sums equal to b and column sums equal to c.

Notice that this second condition did *not* assume a solution of the form we are seeking, that is, derived from ***A*** by scaling the rows and columns. It does, however, have some important implications. The first is that the sum of the elements of b must be the same as the sum of the elements of c. A further implication is that, if it is possible rearrange the rows and columns of ***A*** so that an all-zero block appears, then the corresponding subtotals of b and c must be consistent with those blocks remaining zero while the other cells are positive. For example, if

$$A=\begin{pmatrix}1 & 0\\ 1 & 2\end{pmatrix}$$

then we must also have $b_1 < c_1$ and $b_2 > c_2$. In practice, one insures that the first implied condition (the equality of the sum of row sums and column sums) is met before beginning the RAS calculations. If they fail to converge, then one looks for inconsistencies along the lines of the second implication.

The proof of the convergence of the RAS procedure under these general conditions requires a complicated notation. The essence of the proof, however, can be seen in the special case in which ***A*** is all positive, and we will limit ourselves to that case. (For the general case, see M. Bacharach, *Biproportional Matrices.* Cambridge University Press, 1970.)

We will start the process by scaling the rows, then the columns, and so on. In the first row scaling, we choose the first-round row-scaling factors by

$$r_i^{(1)}=b_i/\sum_j a_{ij} \tag{17.2.1}$$

where the superscript on the r refers to the iteration number. Then we compute the first-round column scaling factors by

$$s_j^{(1)}=c_j/\sum_i r_i^{(1)} a_{ij} \tag{17.2.2}$$

Then we come back to compute the second-round scaling factors,

$$
\begin{aligned}
r_i^{(2)} &= b_i / \sum_j r_i^{(1)} a_{ij} s_j^{(1)} \\
&= b_i / \sum_j \frac{b_i}{\sum_k a_{ik}} a_{ij} s_j^{(1)} \\
&= 1 / \sum_j a_{ij} \frac{s_j^{(1)}}{\sum_k a_{ik}}
\end{aligned}
\tag{17.2.3}
$$

Thus, we can see that the second-round row factors are reciprocals of convex combinations of the first-round column factors, that is, they are reciprocals of a weighted average of those first-round column factors with positive weights which sum to 1. Thus,

$$
\max_i r_i^{(2)} \le 1 / \min_j s_j^{(1)} \quad \text{and} \quad \min_i r_i^{(2)} \ge 1 / \max_j s_j^{(1)} \tag{17.2.4}
$$

By similar reasoning,

$$
\max_j s_j^{(2)} \le 1 / \min_i r_i^{(1)} \quad \text{and} \quad \min_j s_j^{(2)} \ge 1 / \max_i r_i^{(1)} \tag{17.2.5}
$$

The inequalities in 17.2.5 imply 17.2.6

$$
1 / \max_j s_j^{(2)} \ge \min_i r_i^{(1)} \quad \text{and} \quad 1 / \min_j s_j^{(2)} \le \max_i r_i^{(1)} \tag{17.2.6}
$$

Then combining the first inequality of (17.2.4) with the second of (17.2.6) and the second of (17.2.4) with the first of (17.2.6) gives

$$
1 / \min_j s_j^{(1)} \le \max_i r_i^{(1)} \quad \text{and} \quad \min_i r_i^{(2)} \ge 1 / \max_j s_j^{(1)} \ge \min_i r_i^{(1)} \tag{17.2.7}
$$

In other words, the biggest element of r diminishes from iteration to iteration while the smallest rises. Since $\boldsymbol{A}$ is all positive, all of the inequalities in (17.2.4) through (17.2.7) will be strict inequalities unless all the elements of $\boldsymbol{r}$ are equal or all the elements of $\boldsymbol{s}$ are equal. But if they are all equal, they must be all be equal to 1, for otherwise the scaling would increase or decrease the total of all elements in the matrix, contrary to the fact that, after the first row scaling, the sum of all elements remains equal to the common sum of the vectors $\boldsymbol{b}$ and $\boldsymbol{c}$. Since the sequences of $\boldsymbol{r}^{(k)}$ and $\boldsymbol{s}^{(k)}$ vectors both lie in closed, bounded sets, they have limit points. Can these limit points be anything other than the vectors that are all 1's? No, because at any such point, one more iteration of the process would bring a finite reduction of the maximum element (and a finite increase in the minimum element) of each vector. (This is where we use the all positive assumption to have strict inequalities in (17.2.7).)

Thus, for points sufficiently close to these limit points, the next iteration must also bring lower maximal and higher minimal elements than those of the limit point, contrary to the limit point being

a limit point. Therefore the unique limit of each sequence of vectors is a vector of ones.

Thus the convergence is proven for the case of all positive $\boldsymbol{A}$. The proof is similar for $\boldsymbol{A}$ with some 0 elements, but in this case, it may require several iterations to get a finite reduction in the maximal elements of ***r*** and ***s***.

In practice, the condition that the sum of ***b*** equals the sum of ***c*** is checked and assured before the iterative process begins. The initial ***r*** and ***s*** vectors should be reported by the program because they often indicate discrepancies between ***b*** and ***c*** vectors and the initial $\boldsymbol{A}$ matrix. Once the iterations start, the largest and smallest elements of the ***r*** and ***s*** vectors should be reported every five or ten iterations. It is common to observe "wars" between a row control and a column control when one element looms large in both its row and column, but the control totals for the two are quite different. Such "wars" are symptomatic of a failure of the second assumption and an indication that the ***b*** and ***c*** vectors should be revised.

It should be noted that the RAS procedure works for rectangular matrices just as well as for square ones. It is also useful in making input-output tables and the bridge matrices used to convert investment by investor to investment by product bought or consumption by consumer categories to consumption by product categories used for productive categories.

17.3. Preliminary Adjustments Before RAS

It often happens in updating or making tables that one has better information about some cells than about others. For example, in updating the a table with a Glass row, we may have quite good information on the sales of glass products to Beer, because we have information on the production of glass beer bottles. In this case, we can simply remove the "relatively well-known flow" from both its row and column control, perform the RAS balancing on the remaining flows, and then put back in the known flow.

The problem with this procedure is that the "relatively well-known flows" tend to be big flows. If they are not quite consistent with the row or column controls, then removing them requires that all of this inconsistency should be attributed to changes in the remaining small flows. Thus, the small flows can be pushed about rather considerably. This problem can be reduced by a preliminary scaling of the relatively well-know flows before removing them from the process. To describe this adjustment, let R_i be the sum (in the base year) of the relatively well-known flows in row i ; S_i , the sum of the other flows; and B_i , the row control. Then let

$$\alpha = \frac{R_i}{R_i + S_i} \tag{17.3.1}$$

and define z_i as the solution to

$$S_i z_i + R_i z_i^{\alpha} = B_i \tag{17.3.2}$$

The value of z_i which satisfies (17.3.2) is readily found by Newton's method. We then scale all the relatively well-known flows by z_i^{α} and all the other flow by z_i . By (17.3.2), the row will then have the correct sum. By (17.3.1), z_i^{α} is closer to1.0 than is z_i*;* that is to say, the relatively well-known flows are scaled less than the other flows. They are however, scaled somewhat. If they

account for a small fraction of the total of all flows in the row, they will be scaled but little; if they account for much of the row, they will be scaled almost as much as the other flows.

After this preliminary scaling, the known flows can be removed for the rest of the RAS process. While this scaling may seem a bit arbitrary, in practice it has given plausible results in many applications. In fact, it worked so well that the first person working with it, Thomas Reimbold, felt that the *z* must stand for *Zauber*, "magic" in German, his native language. The procedure is therefore often referred to as the *Zauber* process.

CHAPTER 18. TRADE AND TRANSPORTATION MARGINS AND INDIRECT TAXES

18.1. Trade and Transportation Margins

A perennial problem in applied input-output analysis is the treatment of trade and transportation margins and of indirect taxes. The problem is nicely illustrated with transportation costs. If output is valued at the producer's price — the price at the factory gate, so to speak — then the cost of transporting the goods to the user must be considered to be paid separately by the purchasing industry. Thus, the cost of the rail services used in hauling the coal used by electric power plants shows up as an input of rail transportation into electric generation. The cost of hauling generation equipment to and from the utilities' repair facilities would appear in the same cell. Similarly, the cost of hauling coal to a steel mill and of hauling iron ore to the same mill will appear in the same cell.

The problems with this treatment are:

1. It puts quite diverse activities into the same cell; and
2. The table does not reflect the way the rail industry thinks about its business. It thinks in terms of products hauled — and prepares statistics on products hauled, not on industries to which it delivers.

Despite these problems, this treatment is the one most commonly followed. All of the problems apply with equal force to all the other transportation margins and to wholesale and retail trade margins.

One alternative is to change the measure of output of the industry to include the cost of delivering the product to the user. One disadvantage of this treatment is that it removes the numbers in the input-output table one step further from the numbers in terms of which people in the industry think, namely in producer prices. Another problem is that transportation margins may be very different for a dollar's worth of product delivered to different users. The transportation cost of oil delivered to an electric utility by pipeline from a marine terminal may be very different from delivering by truck or rail to a small industrial user.

A better alternative is to add another dimension to the input-output tables. Thus, corresponding to each cell of the tables we have considered so far there would be a vector. The first entry in the vector would be the transaction in producer prices; the second entry would show the rail margin; the third, the truck margin; the fourth, the air freight; and so on through the wholesale and retail trade margins. In effect, we would have a table with layers, the first layer for the producer price transaction, the second for the rail margins, and so on. In fact, the benchmark tables for the United States are prepared with all this information. It has not been commonly used because the size of the matrices involved has been, until fairly recently, large relative to the power of the computers available. That constraint has now been effectively removed, and we may ask, How would we in fact compute with such a layered table?

If $\boldsymbol{A}$ represents the coefficient matrix in producer prices and $\boldsymbol{T_i}$ represents the ith layer of transportation and trade margin coefficients, then the fundamental input-output equations become

$$q = Aq + \sum_i S_i T_i q + f = \left(A + \sum_i S_i T_i\right) q + f \tag{18.1.1}$$

where $\boldsymbol{S_i}$ is a matrix with 1's in the row which produces the service distributed by layer *i* and

elsewhere all zero. The matrix is, in fact, the matrix in producer prices with which it has been traditional to compute. What is gained by distinguishing the layers is not a correction of the traditional computations but rather a better description of what the flows are and a better basis for studying changes in coefficients in the $\boldsymbol{T}_i$ matrices.

18.2. Indirect Taxes, Especially Value Added Taxes

Indirect taxes such as property or franchise taxes are treated as a component of value added, along with depreciation, profits, interest, and labor compensation. Excise taxes such as those on gasoline, alcohol, and tobacco are usually similarly treated, but with less justification, because some uses of these products are exempt. For example, gasoline used to power agricultural machinery or exported whiskey or cigarettes are exempt. Thus, these taxes should also be treated as a layer of the table, since they are not uniform for all cells. Retail sales taxes are usually treated as a component of value added by Retail trade. This treatment assumes that the tax is proportional to the retail margin in all products in all cells. In fact, there are different tax rates on different products, and some products are sold by retail establishments for intermediate use without retail sales tax.

The greatest problems, however, have probably been created by the value added tax (VAT) in the tables of countries which use this tax, a group that now includes all members of the European Union and numerous other countries. Producers pay VAT on the value of their sales but may deduct the VAT paid on their purchases. VAT is not charged on certain products, such as health services or exports. Many European input-output tables have been published in producer prices plus non-deductible VAT. That practice meant that the cell for paper products sold to the hotel industry did not contain VAT, because the VAT on those sales was deductible from the VAT owed by the hotels. The cell for paper products sold to hospitals, however, contained VAT, because the hospitals owed no VAT from which the VAT on the paper products could be deducted. Similarly, since households owe no VAT, they cannot deduct the VAT on the paper products they buy, so the VAT is included in the cell showing the sales of paper products to households. Thus, the cells in the paper products row of such a matrix have very diverse levels of VAT content. That means that the valuation of the product across the row is not homogeneous. It takes more wood pulp to make a dollar's worth paper towels used by a hotel than to make a dollar's worth of paper towels used by a hospital or household, because a significant portion of their dollar goes to VAT. This heterogeneity in the pricing in the row is obviously detrimental to the accuracy of the input-output calculations. The solution to the VAT problem is simply to create a VAT layer of the table.

CHAPTER 19. MAKING PRODUCT TO PRODUCT TABLES

19.1. The Problem

Makers of input-output tables often find data on inputs not by the *product* into which they went but by the *industry* that used them. An *industry* is a collection of establishments with a common principal or *primary* product. But besides this primary product, any one of these establishments may produce a number of *secondary* products, products primary to other industries. Establishments classified in the Cheese industry may also produce ice cream, fluid milk, or even plastic moldings. Consequently, the Cheese industry may have inputs of chocolate, strawberries, sugar, plastic resins, and other ingredients that would appall a connoisseur of cheese. The inputs, however, are designated by what the product was, not by what industry made them. Similarly, data on the final demands, such as exports and personal consumption expenditure, is by product exported or consumed, not by the industry which made it. Thus, input-output matrices usually appear in two parts. The first part, called the Use matrix, has products in its rows but industries in its columns. The entries show the use of each product (in the rows) by each industry (in the columns.) The second part, called the Make matrix, has industries in the rows and products in the columns; the entries show how much of each product was made in each industry. (Some statistical offices also publish instead of the Make matrix a Supply matrix, which is product by industry like the Use matrix, but contains the information in the Make matrix, plus information on margins and taxes on domestic production and imports. This format is now recommended by the *2008 System of National Accounts*).

How can we use these two matrices to compute the outputs of the various products necessary to meet a final demand given in product terms?

One way is to consider that each product will be produced in the various industries in the same proportion as in the base year of the table. This assumption is used, for example, in computable general equilibrium models based on social accounting matrices that explicitly show the Make and Use matrices. This assumption, however, can produce anomalous results. In the above example, an increase in the demand for cheese would automatically and immediately increase demand for chocolate, strawberries, and sugar. These are not common ingredients in most varieties of cheese! There must be a better way to handle the problem.

This highly unsatisfactory situation has led to efforts to make a product-to-product matrix. Indeed, the problem is so well recognized that the "Transmission programme of data" of the European system of accounts requires that all national statistical offices of the member states of the European Union transmit "symmetric" input output tables to Eurostat every five years. No real advice, however, is offered by Eurostat to the statistical offices on how to make these product-to-product tables. This chapter offers a valuable tool for the process. ("Symmetric" is here intended to mean that the same concepts are used in both rows and columns. Its use as applied to these matrices is both highly confusing and not descriptive. Since it is the *nature* of the rows and columns that is the same, not their measure, *symphysic* would be both a better characterization and less confusing.)

To make such a matrix, we need to employ an additional assumption. There are basically two alternatives:

1. The *product-technology assumption*, which supposes that a given product is made with the

same inputs no matter which industry it is made in.

2. The *industry-technology assumption*, which supposes that all products made within an industry are made with the same mix of inputs.

The *System of National Accounts 1993* (SNA) reviews the two assumptions and finds (Section 15.146, p. 367)

> On theoretical grounds, ... the industry technology assumption performs rather poorly" and is highly implausible.

And in the following Section 15.147:

> From the same theoretical point of view, the product (commodity) technology model seems to meet the most desirable properties .... It also appeals to common sense as it is found *a priori* more plausible than the industry technology assumption. While the product technology assumption thus is favoured from a theoretical and common sense viewpoint, it may need some kind of adjustment in practice. The automatic application of this method has often shown results that are unacceptable, insofar as the input-output coefficients appear as extremely improbable or even impossible. There are numerous examples of the method leading to negative coefficients which are clearly nonsensical from an economic point of view.[8]

Since 1967, the Inforum group has used a "semi-automatic" method of making "some kind of adjustment" in calculations based on the product-technology assumption, as called for by the SNA. We have used it with satisfactory results -- and without a single negative coefficient -- on every American table since 1958. The method was published in Almon 1970 and in Almon *et al.* 1974. Despite this long and satisfactory use of the method, it seems not to have come to the attention of the general input-output community. In particular, the authors of the section quoted from the SNA seem to have been unaware of it. This chapter illustrates the method and expands the previous exposition with an example, provides a computer program in the C++ language for executing the method, and presents some of the experience of applying the method to the 1992 table for the USA.

19.2. An Example

An example will help us to visualize the problem. The Table 19.2.1 below shows the Use matrix for a 5-sector economy with a strong concentration in dairy products, especially cheese and ice cream.

We will call this matrix *U*. The use of chocolate in makings cheese and rennet in making ice cream alerts us to the fact that the columns are industries, not products. (Rennet is a substance used to make milk curdle. It is commonly used in making cheese but never in ice cream.)

8 The United Nations *Handbook of Input-Output Table Compilation and Analysis*, 1999 (pp. 98-103) goes into this question in further detail, and also comes out strongly in favor of using the product technology to develop product by product tables. SNA 2008 however, glosses over the topic, and falls back into the stance of arguing that since the product technology assumption will produce negatives, that it is implausible (28.56, p. 515).

Table 19.2.1. The Use Matrix

USE	Industries				
Products	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	0	0	0	0	0
Ice cream	0	0	0	0	0
Chocolate	4	36	0	0	0
Rennet	14	6	0	0	0
Other	28	72	30	5	0

The Make matrix, shown in Table 19.2.2 below, confirms that cheese is being made in the ice cream industry and ice cream in the cheese industry.

Table 19.2.2. The Make Matrix

MAKE	Products				
Industries	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	70	20	0	0	0
Ice cream	30	180	0	0	0
Chocolate	0	0	100	0	0
Rennet	0	0	0	20	0
Other	0	0	0	0	535
Total	100	200	100	20	535

This matrix shows that of the total output of 100 of cheese, 70 was made in the Cheese industry and 30 in the Ice cream industry, while of the total ice cream output of 200, 180 was in the Ice cream industry and 20 in the Cheese industry. It also shows that, of the total output of 90 by the cheese industry, 78 percent (70/90 = .77778) was cheese and 12 percent ice cream. We will need the matrix, *M*, derived from the Make matrix by dividing each cell by the column total. For our example, the *M* matrix is shown in Table 19.2.3.

Table 19.2.3. The *M* Matrix

M	Products				
Industries	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	0.7	0.1	0.0	0.0	0.0
Ice cream	0.3	0.9	0.0	0.0	0.0
Chocolate	0.0	0.0	1.0	0.0	0.0
Rennet	0.0	0.0	0.0	1.0	0.0
Other	0.0	0.0	0.0	0.0	1.0

Now let us suppose that, in fact, cheese is made by the same recipe wherever it is made and ice cream likewise. That is, we will make the "product-technology assumption." If it is true and the

matrices made well, then there exists a "recipe" matrix, R, in which the first column shows the inputs into cheese regardless of where it is made, the second column shows the inputs into ice cream regardless of where it is made, and so on. Now the first column of U, U_1, must be .70*R_1 + 0.10*R_2 ,where R_1 and R_2 are the first and second columns of R, respectively. Why? Because the Cheese plants make 70 percent of the cheese and ten percent of the ice cream. In general,

$$U = RM' \tag{19.2.1}$$

Where M' is the transpose of M. It is then a simple matter to compute R as

$$R = U(M')^{-1} \tag{19.2.2}$$

For our example, $(M')^{-1}$ is given in table 19.2.4.

Table 19.2.4 M' Inverse

	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	1.5	-0.5	0.0	0.0	0.0
Ice cream	-0.2	1.2	0.0	0.0	0.0
Chocolate	0.0	0.0	1.0	0.0	0.0
Rennet	0.0	0.0	0.0	1.0	0.0
Other	0.0	0.0	0.0	0.0	1.0

and R works out to be

Table 19.2.5 The *R* or "Recipe" Matrix

R	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	0	0	0	0	0
Ice cream	0	0	0	0	0
Chocolate	0	40	0	0	0
Rennet	20	0	0	0	0
Other	30	70	30	5	0

This R is very neat. All the rennet goes into cheese and all the chocolate goes into ice cream. Unfortunately, as indicated by the quotation from the SNA, it is rare for the results to turn out so nicely.

Indeed, just a slight change in the U matrix will show us what generally happens. Suppose that the U matrix had been just slightly different, with 1 unit less of chocolate going into cheese as shown below and one less unit of rennet used in ice cream.

Table 19.2.6 An Alternative Use Matrix

Alternative USE	Industries				
Products	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	0	0	0	0	0
Ice cream	0	0	0	0	0
Chocolate	3	37	0	0	0
Rennet	15	5	0	0	0
Other	28	72	30	5	0

Table 19.2.7 shows what the *R* matrix would have been:

Table 19.2.7 An Impossible *R* Matrix

Impossible R	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	0.0	0.0	0.0	0.0	0.0
Ice cream	0.0	0.0	0.0	0.0	0.0
Chocolate	-1.7	41.7	0.0	0.0	0.0
Rennet	21.7	-1.7	0.0	0.0	0.0
Other	30.0	70.0	30.0	5.0	0.0

Here we find the infamous small negative flows. It is not hard to see how they arise. While it is conceivable that the Cheese industry does not produce chocolate ice cream, it is also very easy for the table makers to forget to put into the Cheese industry the chocolate necessary for the ice cream it produces, or to put in too little. Wherever that happens, negatives will show up in the *R* matrix.

The negatives have driven at least some statistical offices to the industry-technology assumption. The so-called commodity-to-commodity matrix, *C*, derived from this assumption is

$$C = UN' \tag{19.2.3}$$

where *N* is the matrix derived from the Make matrix by dividing each row by the row total. For example, the Cheese column of *C* is $C_1 = .77778U_1 + 0.14285U_2$ because 77.778 percent of the product of the first industry is cheese and 14.285 percent of the product of the second industry is cheese. The result of applying this assumption to our example is Table 19.2.8.

Table 19.2.8 The Mess Made by the Industry Technology Assumption

C Industry Technology	Industries				
Products	Cheese	Ice cream	Chocolate	Rennet	Other
Cheese	0	0	0	0	0
Ice cream	0	0	0	0	0
Chocolate	8.254	31.746	0	0	0
Rennet	11.746	8.254	0	0	0
Other	32.063	67.936	30	5	0

This "solution" has made matters worse. The original *U* matrix had 4 units of chocolate going into the Cheese industry, which admittedly made some ice cream. Now this industry-technology product-to-product matrix asserts that *8.25 units of chocolate went into producing pure cheese*! Not into the Cheese industry but into the *product* cheese! And 8.25 units of rennet went into producing curdled ice cream! To call the result a product-to-product table would be little short of scandalous.

Fortunately, we do not have to choose between this sort of massive nonsense and negative flows. It is perfectly easy to rely mainly on the product-technology assumption, yet avoid the negatives, as we will now show.

19.3. The No-Negatives Product-Technology Algorithm

We wrote the basic equation relating *U*, *M*, and *R* as equation (19.2.1) above. It will prove convenient to rewrite equation (19.2.1) as

$$U' = MR' \tag{19.3.1}$$

Using U_i' to denote the i^{th} column of U' and R_i' to denote the i^{th} column of *R*, we can write

$$U_i' = MR_i' \tag{19.3.2}$$

Notice that this is an equation for the distribution of product i in row i of the Use matrix as a function of *M* and distribution of the same product in row i of the *R* matrix. We can simplify the notation by writing

$$u = U_i' \quad \text{and} \quad r = R_i' \tag{19.3.3}$$

Then equation (19.3.2) becomes

$$u = Mr \tag{19.3.4}$$

or

$$0 = -Mr + u \tag{19.3.5}$$

And adding *r* to both sides gives

$$r = (I - M)r + u \tag{19.3.6}$$

Apart from the unusual case in which less than half of the production of a product is in its primary industry, the column sums of the absolute values of the elements of (*I* - *M*) are less than 1, and the convergence of the Seidel iterative process for solving this equation is guaranteed a by well-known theorem. (If the share of the total production of a particular product coming from the industry to which it is primary is *x*, then the absolute value of the diagonal of (*I* - *M*) for that product is $|1 - x|$ and the sum of all the absolute values of off-diagonal elements in the column is $|1 - x|$, so the total for the column is $2*|1 - x|$, which is less than 1 if $x > .5$.)

We start this process with

$$r^{(0)} = u \tag{19.3.7}$$

and then define successive approximations by

$$r^{(k+1)}=(I-M)r^{(k)}+u \tag{19.3.8}$$

To see the economic interpretation of this equation, let us write out the equation for the use of a product, say chocolate, in producing product *j*, say cheese:

$$r^{(k+1)}=u_j-\sum_{h=1;\,h\neq j}^{n} m_{jh}r_h^{(k)}+(1-m_{jj})r_j^{(k)} \tag{19.3.9}$$

The first term on the right tells us to begin with the chocolate purchases by the establishments in the cheese industry. The second term directs us to remove the amounts of chocolate needed for making the secondary products of those establishments by using our present estimate of the technology used for making those products, $r^{(k)}$. Finally, the last term causes us to add back the chocolate used in making cheese in other industries. The amount of chocolate added by the third term is exactly equal to the amount stolen, via second terms, from other industries on account of their production of product *j*:

$$(1-m_{jj})r_j^{(k)}=\sum_{h=1;\,h\neq j}^{n} m_{hj}r_j^{(k)} \tag{19.3.10}$$

because

$$\sum_{h=1}^{n} m_{hj}=1 \tag{19.3.11}$$

It is now clear how to keep the negative elements out of *r*. When the "removal" term, the second on the right of (19.3.9), is larger than the entry in the Use matrix from which it is being removed, we just scale down all components of the removal term to leave a zero balance. Then instead of adding back the "total-stolen-from-other-industries" term, $(1-m_{jj})r_j$, all at once, we add it back bit-by-bit as it is captured. If a plundered industry, say Cheese, runs out of chocolate with only half of the total chocolate claims on it satisfied, we simply add only half of each plundering product's claim into that product's chocolate cell in the *R* matrix. We will call the situation where the plundered industry runs out of the product being removed before all claims are satisfied a "stop".

To express this process in equations, we introduce scale factors, $s_j^{(k)}$, defined by

$$s_j^{(k)}=1 \quad \text{if} \quad u_j\geq\sum_{h=1;\,j\neq j}^{n} m_{hj}r_j^{(k)} \tag{19.3.12}$$

and

$$s_j^{(k)}=\frac{u_j}{\sum_{h=1;\,h\neq j}^{n} m_{hj}r_j^{(k)}} \quad \text{otherwise} \tag{19.3.13}$$

Equation (19.3.9), which expresses the Seidel process without the no-negatives condition, is then replaced by

$$r^{(k+1)}=u_j-s_j^{(k)}\sum_{h=1;\,h\neq j}^{n} m_{jh}r_h^{(k)}+\sum_{h=1;\,h\neq j}^{n} s_h^{(k)}m_{hj}r_j^{(k)} \tag{19.3.14}$$

By the choice of the scale factors, **s**, we are sure that $r_j^{(k+1)}$ is not negative. By summing both sides of (19.3.14) over *j*, it is easy to see that

$$\sum_{j=1}^{n} r_j^{(k+1)} = \sum_{j=1}^{n} u_j \qquad (19.3.15)$$

That is to say, the row sum is unchanged by the iterative process. In the computer program, it should be pointed out, there is no need for a vector, **s**, of scale factors. Instead, a vector combining the second and third terms is built up as each scale factor is calculated. In this way, the multiplications do not have to be done twice.

The process can also be applied to the rows of the value added part of the matrix. It is not certain, however, that the column sums of the resulting value-added table will match the value added as calculated from product output minus intermediate input. An option on the program provides for the automatic RAS balancing at the end of the no-negatives algorithm to ensure that the resulting matrix has matching row and column totals.

19.4. When Is It Appropriate to Use This Algorithm?

This algorithm is appropriate where the product-technology assumption itself is at least approximately true. Essentially, it allows there to have been slightly different technologies in industries where assuming strictly the average product technology would produce negatives. It is appropriate where the negatives arise because of inexactness in making the tables or because of slight differences in technologies in different industries. Applied to the Use matrix of either Table 1 or Table 6, this method gives the "neat" Recipe matrix of Table 5 with no rennet in ice cream and no chocolate in cheese. It never produces negative entries nor positive entries where Use has a zero. The row totals are unaffected by the process. It is, moreover, equivalent to deriving Recipe from equation (19.2.1) if no negatives would arise, so that if the product-technology assumption is strictly consistent with the Use and Make tables, the method produces the true matrix. It may even produce a correct Recipe matrix from a faulty Use matrix — as it has perhaps done in our example — so that equation (19.2.1) could be used to revise the estimate of the Use matrix.

Certain accounting practices, however, may produce situations which appear to be incompatible with the product-technology assumption, even though the underlying reality is quite compatible. For example, local electric utilities generally buy electricity and distribute it. In the U.S. tables, they are shown as buying electricity (not coal), adding a few intermediate inputs and labor, and producing only a secondary product, electricity, which is transferred, via the Make matrix, back to electricity. Looked at mechanically, this method of making electricity is radically different from that used in the Electricity industry, which uses coal, oil, and gas to make electricity, not electricity itself. If our algorithm is applied thoughtlessly to this situation, it cannot be expected to give very sensible results.

Fortunately, it is easy to generate signs of this sort of problem. One can compute the “NewUse” matrix implied by equation (19.2.1) with the Recipe matrix found by the algorithm and the given Make matrix. This "NewUse" matrix can then be compared with the original Use matrix and the causes of the differences investigated. We will follow this procedure in section 19.6 on the experience of using the method on the 1992 tables for the USA.

To fix the problem in the above example about electricity, we have only to consider the output of the State and local utilities as production of their own primary product, which is then sold, via the Use matrix — not transferred via the Make matrix — to the Electricity industry. In essence, we use the industry technology assumption for the local electric utilities — and for all other industries where all of the output is secondary. The industry technology assumption may also be preferable for transfers to some catch-all sectors such as "Miscellaneous food preparations" (SIC2099), which includes such disparate products as vinegar, yeast, Chinese noodles, and peanut butter. It is probably just as reasonable to suppose that a product transferred into this industry is made with the average technology of the industry where it is made as with the average technology of this catchall sector. Indeed, this sort of industry can produce the reverse of the negatives problem. For example, because of the importance of peanut butter in this industry, it has significant inputs of oil seeds. Now the no-negatives algorithm will not pull oil seeds out of the "Macaroni, spaghetti, vermicelli, and noodles" industry, (SIC2098), (which used no oil seeds) just because it transferred some Chinese noodles to 2099. But neither will it take out an adequate amount of flour for those noodles, because flour is quite unimportant in the 2099 input mix. This problem shows up only indirectly by substantial oil-seed inputs to many food industries in the NewUse matrix which transferred products to 2099 but, in fact, used no oil seeds. That is a signal to switch to the industry technology for these transfers by converting them to sales in the Use matrix.

Thus, in the use of this method, a number of iterations may be necessary. Changes in concepts, in treatments of some transactions, and occasionally in underlying data may be necessary. Although the calculation of the non-negative Recipe matrix is totally automatic, it may be necessary to make several runs to get acceptable results.

In this process, it must be recognized that a nice, clean accounting system may not be operational, that is, it may not provide by itself a simple, automatic way to go from final demand vectors specified by products to total outputs of those products. We may have to change slightly some of the concepts in the accounting system to make it operational. In making the change required for the Electricity example, we have messed up the neat accounting concept of the Electricity column of the Use matrix as a picture of what came into a particular group of establishments. We have, however, taken a step toward creating what might be called an operational Use matrix. I do not say, therefore, that statistical offices should not produce pure accounting Use matrices. But I do feel that they should also prepare the operational use matrix and the final product-to-product matrix, for in the process, they will learn about and deal with the problems which the users of the matrix will certainly encounter. They may even discover and correct errors in their work before they are discovered by their users.

This process is totally inappropriate for handling by-products such as hides produced in the meat packing industry or metal scrap produced in machinery industries. Their treatment is a different subject.

19.5. A Brief History of the Negatives Problem

The idea to compute *R* from equation (19.2.1) seems to have been first put in print by Van Rijckeghem (1967). He realized that there could be negatives but did not think they would be a serious problem. The idea of using equation (19.2.1) in this way, however, must have been in the air, for by early 1967, I had used it, without thinking that it was original, found negatives, and started

work on the algorithm presented here.

The problem was encountered by ten Raa, Chakraborty and Small [1984] in the course of work which was primarily concerned with identifying by statistical means true by-products. They note the existence of the method presented here but write:

> [Almon] iterates truncated Neumann series in which matrix multiplications are carried out only to a limited extent to avoid negatives. This arithmetic manipulation goes without justification, is arbitrary and depends on the choice of [make matrix]-decomposition as well as the iteration scheme.

I do not believe that any of this comment is correct. The Neumann series is the expansion $(I-A)^{(-1)}=I+A+A^2+A^3+...$. The algorithm used here makes no use of this series; rather it uses the Seidel procedure. There are no matrix multiplications, nor is there is any equivalence between a "limited" number of terms in the Neuman series and the Seidel solution. The procedure is carried to convergence. We have seen that the procedure has a perfectly reasonable economic interpretation; indeed, it arose from the economic interpretation of the Seidel procedure. The only thing perhaps "arbitrary" is that 0 is considered a reasonable input flow while negatives are considered unreasonable. I do not know what the "[make matrix]-decomposition" refers to, but I can assure the reader that the solution does not depend on the "iteration scheme." While I could not see how it could, given that it is carried to convergence, I changed the program and ran the "robberies" in the opposite order. The answers were identical.

The ingenious attempt of ten Raa [1988] to modify elements of the matrices in such a way as to find a most probable *U* matrix consistent with a non-negative *R* should be mentioned even though it ended, in the author's view, in frustration.

Rainer and Richter [1992] have documented a number of steps which they took towards making what I have called here the operational Use and Make matrices. Such steps should certainly be considered and applied if need. These authors still ended up with hundreds of negative flows in the *R* matrix because they were using just equation (1). At that point, the process described here could have been applied.

Steenge and Konijin [1992] point out that if the *R* matrix computed from equation (19.2.1) has any negatives in it, then it is possible to change the levels of output of the various industries in such a way that more of all products is produced without using more of all inputs. They feel that it is implausible that such a rearrangement is possible and observe that perhaps the negatives "should not be regarded as rejecting the commodity technology assumption, but as indicators of flaws in the make and use tables." (p. 130). I feel that there is much merit in that comment. It seems to me that the right time and place to use the algorithm presented here is in the process of making the tables. If there are not good statistical grounds for preferring the original Use matrix, the recomputed NewUse might well be argued – following the reasoning of Steenge and Konijn – to be a better estimate.

The caveat here is that there may well be cases where it really would be possible to increase the outputs of all products while using less of some product. For example, if there are shoes made in the Plastics products industry without any use of leather, while the Footwear industry uses leather, then by moving shoe production from Footwear to Plastic products it may be possible to produce more of all products while using less leather. Where such cases arise, a different solution is necessary, for example, moving the shoes made in the Plastics products industry together with their inputs into the

Footwear industry or insisting that the two kinds of shoes are separate if substitutible products.

19.6. Application to the U.S.A Tables for 1992

The method described here has been applied to all of the USA tables since 1958 with experiences broadly similar to those described here for the 1992 table. This table has 534 sectors, counting some construction sectors which have no intermediate sales. Of these 534, 425 have secondary production. Of the 283,156 possible cells in a 534 X 534 matrix, the Use matrix has 44,900 non-zero cells, and the Make matrix has 5,885. The matrix was produced in two versions. In one, certain activities, such as restaurant services of hotels, were removed from the industry where they were produced (Hotels) and put into the sector where these activities were primary (Restaurants). In the other, these activities were left in the industry where they were conducted. The first version was designed to make the product-technology assumption more valid, and it has been used here. The matrix also puts true by-products (such as hides from meat packing) in a separate row, not one of the 534 considered here.

To try to convey a feeling of what it is like to work with the algorithm, we will look at the process midway along, rather than at the very beginning or the somewhat polished end. That is, some adjustments in the Use and Make matrix from which the algorithm starts will have already been made. As a result of this application, further adjustments will be suggested before the next application.

Before this application of the algorithm, the output of industries which had only secondary production had been changed, for reasons explained above, to be primary and the flows moved from the Make to the Use matrix.

In the following rather detailed descriptions, necessary to give a picture of what the process is really like, I will, to avoid confusion, capitalize the first letter of the first word in industry names but not in product names.

The industry Water and sewer systems failed to satisfy the requirement that at least half of the output of a product should be in the industry where it is primary. Indeed, some 85 percent of this product's output comes from Other state and local enterprises, and the iterative procedure failed to converge for a few rows until this secondary transfer was converted into a primary sale. Production of secondary advertising services, which occurred in many sectors, was also converted to a primary product of the producing industry and "sold" via the Use matrix to the Advertising industry. Secondary production of recreational services in agricultural industries was similarly converted. Much of the output of the several knitting industries had been treated originally as secondary production, and these had been changed to primary sales before the calculations shown here. Finally, the diagonals of many columns of the Use matrix are large, in part because intra-firm services, such as those of the central offices, often appear there. Thus the same sort of service that is on the diagonal of industry i is also on the diagonal of industry j. In this case, the product-technology assumption does not apply, not because it is untrue, but because of the way the table was made. Until we are able to obtain tables without this problem, we have just removed half of the diagonals from the Use table before calculating Recipe, and have then put back this amount in both of these matrices and in the NewUse matrix.

The data in both Use and Make tables were given to the nearest 1 million dollars, and all dollar

figures cited here are in millions. The convergence test in the iterative process was set at one tenth of that amount, .1 million dollars. The iterative process converged for most rows of the *R* matrix in less than five iterations. The most iterations required for any row was 15.

The resulting Recipe matrix looks very similar in most cells to the original Use table. The Recipe matrix contains, of course, only non-negative entries and can have strictly positive entries only where *U* has positive entries. It may, however, as a result of the "robbing" process, have a zero where *U* has a positive entry. In all, there were only 95 cells in which Recipe had a zero where Use had a positive entry.

Although it is the Recipe matrix that we need from this process, it is also interesting, as noted above, to compare the original Use matrix with what we may call NewUse, computed by the equation (19.2.1) by NewUse = Recipe*Make'. The difference between Use and NewUse shows the changes in the Use matrix necessary to make it strictly compatible with product-technology assumption, the given Make matrix, and the calculated Recipe matrix. If there was no "stop" in a row, the two matrices will be identical in that row. There were 118 such identical rows, 109 of them having no secondary output.

In the other rows, these differences turn out to be mostly small but very numerous. The first and most striking difference is that NewUse has almost twice as many non-zero cells as does Use. Nearly all of these extra non-zeros are very small, exactly the sort of thing to be reasonably ignored in the process of making a table. But it is precisely this "reasonable ignoring" that leads to the problem of many small negatives in the product-to-product tables calculated without the no-negatives algorithm.

To get a closer look at how Use and NewUse compare, we may first divide each column by the corresponding industry output and then look at the column sums of the absolute values of the differences of individual coefficients in the column. This comparison is shown in Table 19.6.1. Clearly the vast majority of industries show only small differences compatible with "reasonable ignoring" of small flows in the Use matrix. They, therefore, cast no serious doubt on the product-technology assumption or the usability of the Recipe matrix obtained by the no-negatives algorithm. If what we are interested in is the *R* matrix, we can ignore the small differences between Use and NewUse.

Table 19.6.1 Comparison of Use and NewUse

Sum of Absolute Differences	Count
.050 - .250	17
.030 - .050	24
.020 - .030	54
.010 - .020	117
.000 - .010	312

There are, however, a few cases that should be looked at more closely. Table 19.6.2 shows a list of all of industries which had a sum of absolute differences greater than .050. We will look at the top five.

Table 19.6.2 Largest Differences Between Use and NewUse

Sum |dif|	Column Number	Column Name	Largest single difference Row	|dif|	Row Name
0.250	272	Asbestos products	31	0.023	Misc. nonmetallic minerals
0.232	88	Sausages	3	0.151	Meat animals
0.167	125	Vegetable oil mills, nec	15	0.074	Oil bearing crops incl s
0.118	493	Auto rental & leasing	232	0.025	Petroleum refining
0.088	128	Edible fats and oils, nec	15	0.043	Oil bearing crops incl s
0.088	126	Animal & marine fats	126	0.038	Animal & marine fats &
0.086	87	Meat packing plants	3	0.057	Meat animals
0.079	285	Primary metals, nec	22	0.006	Iron & ferroalloy ore m
0.079	225	Manmade organic fibers	212	0.036	Indl chem: inorg & org
0.074	450	Transportation services	232	0.019	Petroleum refining
0.068	123	Cottonseed oil mills	5	0.048	Cotton
0.065	357	Carburetors, pistons,	391	0.011	Electronic components
0.060	99	Pickles, sauces	1	0.011	Dairy farm products
0.060	95	Canned & cured sea food	19	0.039	Commercial fishing
0.059	139	Yarn mills & textile fini	212	0.035	Indl chem: inorg & org
0.055	459	Sanitary services, steam	413	0.018	Mechanical measuring devices
0.051	248	Leather gloves	244	0.012	Leather tanning

There are, however, a few cases that should be looked at more closely. Table 19.6.2 shows a list of all of industries which had a sum of absolute differences greater than .050. We will look at the top five.

For Asbestos products, the cause of the difference is quickly found. The fundamental raw material for these products comes from industry 31 Misc. non-metallic minerals. Over forty percent of the output of Asbestos products, however, is produced in industry 400 Motor vehicle parts and accessories, but this industry buys neither miscellaneous non-metallic minerals nor asbestos products. In other words, it seems to be making almost half of the asbestos products without any visible source of asbestos. This anomaly seems to me to be an oversight in making the Use matrix which should be simply corrected. If our only interest is the Recipe matrix, the algorithm seems to have computed pretty nearly the right result from the wrong data. On the other hand, if we want to correct the Use table, NewUse, gets us started with the right entry for Misc. non-metallic minerals into both Motor vehicle parts and Asbestos products. To keep the right totals in these two columns of Use will require manual adjustments.

The second largest difference between Use and NewUse shown in Table 19.6.2 is in the input of meat animals into Sausage. The Sausage industry is shown in the Use matrix to buy both animals ($655) and slaughtered meat ($9688). It had a primary output of $13458 and a secondary output of $2612 of products primary to Meat packing. Meat packing had a secondary output of $4349 of sausage. Now in Meat packing, the cost of the animals is over eighty percent of the value of the finished product, so the purchases of animals in the Sausage industry is insufficient to cover even the secondary meat output of this industry, not to mention making any sausage. In making Recipe, the input of animals directly into sausage is driven to zero and cut off there rather than being allowed to become negative. Then when NewUse is made, the direct animal input for all the secondary production of meat packing products is put in, thus making a flow some six times as large as the purchase of meat animals by the Sausage industry in the original Use matrix.

What I believe to be really happening here is that Sausage plants are mostly buying halves of slaughtered animals from meat packers, selling off the best cuts as a secondary product, and using

the rest to make sausage. Over in the Meat packing plants, the same thing is happening. Fundamentally, there is only one process of sausage making. The question is how to represent it in the input-output framework. The simplest representation of it in the Use matrix would be to have packing houses sell to sausage plants only the meat that would be directly used in sausage. The rest, the choice cuts sold off as meat by Sausage mills, would simply be considered sold by the packers without ever passing through the Sausage mills. The industry output of Sausage mills is reduced but cost of materials (namely, meat) is reduced by exactly the same amount, so there is no need to adjust other flows. Product output of meat is reduced, but not the industry output. Thus, a slight adjustment in the accounting makes it broadly compatible with the product-technology assumption. The seventh item in Table 19.6.2, by the way, is just the other side of this problem.

The third largest of the discrepancies lies in row 16, oil-bearing crops, of industry 125 Vegetable oil mills n.e.c (not elsewhere classified). The differences in the underlying flows is not large, $298 in Use and $251 in NewUse, but it turns up in Table 19.6.2 because the cost of these oil crops is such a large fraction of the output of the Vegetable oil mills. A comparison of the oil-bearing crops row of Use and NewUse shows that NewUse has a number of small positive entries for industries where, as for Cheese, Use has a zero and where, moreover, it is highly implausible that there was any use of oil seeds. On the other hand, most of the large users of oil seeds, like Vegetable oil mills have had their usage trimmed back. The key to what is going on is found in industry 132 Food preparations n.e.c.. In Use, this industry bought $558 from oil bearing crops, nearly twice the consumption of the vegetable oil mills themselves. Peanut butter, as noted above, is in this catchall industry. That fact, by itself, is not a problem. The problem is that about a quarter of the production of products primary to this industry are made in other industries. In fact, most of the food manufacturing industries have some secondary production of the miscellaneous food preparations. Probably "preparations" made in the Cheese industry are quite different from those made in the Pickles industry. And it certainly makes no sense to spread oil seed inputs all over the food industries. Here we have a clear case of the inapplicability of the product-technology assumption if all these secondary products are considered to be truly the same product. On the other hand, as argued above, the very heterogeneity of the products makes it appropriate to consider each as a primary product of the industry which produces it and then "sell" it, via the Use matrix, to Food preparations for distribution. In the next pass at making Recipe, this change is to be made.

The vegetable oil industries also present another interesting case of apparent but perhaps not real violation of the product-technology assumption, which shows up in the fifth item in Table 19.6.2. Industry 125 Vegetable oil mills n.e.c. has inputs of oil-bearing crops, cotton, and tree nuts totaling $437. It uses these oil sources to produce a primary output of $572. Industry 128 “Edible fats and oils” produces $92 of products primary to 125 without a penny of any of these inputs! Surely this is flat violation of the product-technology assumption. But is it really? "Edible fats and oils" buys lots of the products primary to Vegetable oil mills. Thus, it is entirely possible to have two bottles of chemically identical oil made of identical raw materials by identical refining processes but with one bottle made entirely in Vegetable oil mills while the oil in the other bottle was pressed in those mills and then sold to Edible fats and oils for finishing. We might call this situation “trans-market product technology.” Our algorithm gave the right answer for the Vegetable oil mills column of Recipe, that is, it combined output of products primary to the oil mills with the inputs of oil sources which this industry had.

The fourth largest discrepancy in Table 19.6.2 is for the gasoline input into Automobile renting and

leasing. Use shows $1131; Recipe ups that to $1197.2; but NewUse cuts it back to $565.5. What happened? The problem is that slightly more than half of the output Auto renting is produced in Credit agencies, with a minuscule input of gasoline. When NewUse is made, more than half of the gasoline in Recipe is allocated over to Credit agencies. Here we are confronted with a failure of the product-technology assumption not because of different processes for producing the same product but because two quite different products have been called one and the same in the accounting system. The output of the Credit agencies, long-term leasing, is quite distinct from the short-term renting, which is were the gasoline was used. The best solution would be to recognize the difference of the two products. Short of that, the worst of the problem can be fixed by turning the secondary transfer from Credit agencies to Automobile rental into a primary flow. The present Recipe matrix, incidentally, is about right in the gasoline row but makes no connection between a final demand for automobile renting and leasing and the output of credit agencies.

From these five or six cases, we see that our algorithm cannot be expected to give usable results on the first try. The problems are likely to lie, however, neither in the fundamental economic reality nor in the algorithm, but in an accounting system which needs a few modifications in Use and Make to make it operational in our sense. Most importantly, the algorithm gives us the means to identify the places that need attention and a way of progressing systematically through the problems. It also provides a way of producing a final, non-negative Recipe matrix that implies a NewUse matrix close enough to the modified Use matrix that the differences can be safely ignored.

Making an input-output table requires fussing over details, and making a good Recipe matrix with the algorithm presented here is no different in this respect from any other part of the process. Use of the algorithm reveals and pinpoints problems. Moreover, the important problems are likely to be small in number. We have covered all of those causing a difference of as much as .100 between columns of Use and NewUse. To get to a Recipe table we would be ready to accept might require another week's work. But in the total effort which went into making this table, that is minuscule. Most importantly, the use of the algorithm gives us a way to work on the problems rather than just wring our hands over negatives.

In this sense, this algorithm has performed satisfactorily over many years on every U.S. table since 1958. The use of the method seems to me to deserve to become a standard part of making input-output tables and, in particular, for making product-to-product tables.

19.7. The Computer Program

The C++ code for this algorithm, using functions from *BUMP*, the Beginner's Understandable Matrix Package, for handling matrices and vectors, is given below. It is reproduced here because the code shows more clearly than the verbal or formulaic description exactly what is done. The program and the supporting *BUMP* code made be downloaded from the Inforum Internet site: www.inforum.umd.edu. The main program here reads in the matrices that were used in the examples. The main program for the actual calculations of the full-scale American matrices is significantly larger and has various diagnostic output, such as that shown in Table 19.6.2. It is available on request.

In using the algorithm, it is important for documenting what has been done to have a method of input of the original Use and Matrix matrices that preserves the original version at the top of the input file and introduces the modifications as over-rides later in the file. It is also important to have software,

such as *ViewMat*, which will show corresponding columns of several large matrices side-by-side in a scrolling grid. *ViewMat* is also available on the Inforum Internet site.

```
#include <stdio.h>      // for printf();
#include <math.h>    // for abs()
#include "bump.h"
int purify(Matrix& R, Matrix& U, Matrix& M, float toler);

void main(){
   Matrix Use(5,5), Make(5,5), R(5,5), NewUse(5,5);
   Use.ReadA("Use.dat");
   Make.ReadA("Make.dat");
   purify(R,Use,Make,.000001);
   R.Display("This is R");
   writemat(R,"Recipe");
   NewUse = R*(~NewUse);
   writemat(NewUse,"NewUse");
   tap();
   printf("\nEnd of calculations.\n");
   }

/* Purification produces a product-to-product (or Recipe) matrix R from a Use matrix U and a Make
   matrix M.  M(i,j) shows the fraction of product j made in industry i.  U(i,j) shows the amount of
   product i used in industry j.  The product-technology assumption leads us to expect that there
   exits a matrix R such that U = RM'.  If, however, we compute R = U*Inv(M') we often find many
   small negative elements in  R.  This routine avoids those small negatives in an iterative process.
*/

int purify(Matrix& R, Matrix& U, Matrix& M, float toler){
   int row, i, j, m, n, iter, imax;
   const maxiter = 20;
   float sum,rob,scale,dismax,dis;
   n = U.rows();  // n = number of rows in U
   m = U.columns(); // m = number of columns in U
   Vector C(m), P(m), Flow(m), Discrep(m);
   // Flow is row of U matrix and remains unchanged.
   // P becomes the row of the purified matrix.
   // C is the change vector at each iteration.
   // At the end of each iteration we set P = Flow + C, to start // the next iteration.

   // Purify one row at a time
   for(row = 1; row <= n; row++){
      C.set(0.); // C, which will receive the changes, is
      // initialized to zero.
      // P = Flow + C will be the new P.
      pulloutrow(Flow,U,row);
      P = Flow;
      iter = 0;
      start: iter++;
      for(j = 1; j<=m; j++){
         // Calculate total claims from other industries on
         // the inputs into industry j.
         sum = 0;
         for(i = 1; i <= m; i++){
            if(i == j) continue;
            rob = P[i]*M(j,i);
            sum += rob;
            C[i] += rob;
            }
         // Did we steal more from j than j had?
         if (sum > Flow[j] && sum > 0){
            // scale down robbery
            scale = 1. - Flow[j]/sum;
            for(i = 1; i <= m; i++){
               if(i == j) continue;
               C[i] -= scale*P[i]*M(j,i);
```

```
               }
            sum = Flow[j];
            }
         C[j] -= sum;
         }
      // Check for convergence
      imax = 0;
      dismax = 0;
      for(i = 1; i <= m; i++){
         dis = fabs(P[i] - Flow[i] - C[i]);
         Discrep[i] = dis;
         if(dis >= dismax){
         imax = i;
         dismax = dis;
         }
      }
   P = Flow + C;
   C.set(0);
   if(dismax > toler){
      if(iter < maxiter) goto start;
      printf("Purify did not converge for row %d. Dismax = %7.2f. Imax = %d.\n",
         row,dismax,imax);
      }
   putinrow(P,R,row);
   }
return(OK);
}
```

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CHAPTER 20. A PERHAPS ADEQUATE DEMAND SYSTEM

Long-term, multisectoral modeling requires calculation of consumer expenditures in some detail by product. Finding a functional form to represent the market demand functions of consumers for this work has proven a surprisingly thorny problem. Clearly, the form must deal with significant growth in real income, the effects of demographic and other trends, and changes in relative prices. Both complementarity and substitution should be possible among the different goods. Increasing income should certainly not necessarily, by the form of the function, force the demand for some good to go negative. Prices should affect the marginal propensity to consume with respect to income, and the extent of that influence should be an empirical question, not one decided by the form of the function.

This chapter will present a form which meets these requirements and extends a form I suggested many years ago (Almon [1979]). Applications of the form to forty-product demand systems for France, Italy, Spain and the United States are reported and the results compared.

Before presenting this form, however, it may be well to see just how tricky it can be to find a form with these simple requirements by looking at another form, the "Almost Ideal Demand System" (AIDS) suggested by Deaton and Muellbauer [1980]. Its name, the eminence of its authors and its place of publication have led to wide usage. It has, however, a most peculiar property which is likely to utterly vitiate any growth model in which it is used. Like many others, it is derived from utility maximization; its problems will therefore emphasize the important fact that such derivation does not automatically imply reasonable properties. One of the properties it does imply, however, is Slutsky symmetry in the market demand functions. This property was not mentioned above. Should it have been? What role should this symmetry play in market demand functions? His questions also needs to be examined before presenting the new form, for it plays a key role its formulation.

20.1. Problems and Lessons of the AIDS Form

The AIDS form can be written as an equation for the budget share of good *i*:

$$s_i = a_i + \sum_{j=1}^{n} d_{ij} \log(p_j) + b_i \log(y/P) \tag{20.1.1}$$

where s_i is the budget share of product *i*, p_j is the price of product *j*, *y* is nominal income and *P* is an overall price index, the matrix of *d*'s is symmetric and has zero row and column sums, the sum of all the a_i is one, and the b_i sum to zero. Consequently, if any b_i is positive, then one or more others must be negative. Thus *increasing real income must ultimately drive the consumption of one or more goods negative*, unless, of course, it has no effect at all on budget shares. This property seems rather less than "ideal". Moreover, the partial derivative of the share with respect to real income is independent of the relative prices, whereas common sense suggests that it should depend on them. Because of these properties, the AIDS form, while possible "almost ideal" from some point of view, is surely absolutely inadequate for use in any growth model. Since it is derived from utility maximization, it also serves as a clear warning that the mere fact of such ancestry is no assurance whatsoever of the adequacy of the form, a lesson which has been heeded in the PADS form proposed here.

A number of other forms derived from utility maximization were reviewed in the article cited and

found wanting relative to the simple properties set out above. The only study which to my knowledge has estimated these forms, AIDS, and the Almon form all on the same data and compared the results is Gauyacq [1985]. Using French data for 1959-1979, he estimated "the linear expenditure system of Stone; the model with real prices and income of Fourgeaud and Nataf; the additive quadratic model of Houthakker and Taylor; the logarithmically additive model of Houthakker, … the Rotterdam model of Theil and Barten, the Translog model based on a logarithmic transformation of the utility function; the AIDS model of Deaton and Muellbauer; ….[and] the model proposed by Clopper Almon." The conclusion was not surprising to anyone who had compared the properties of the forms to the simple requirements stated above: "De l'étude que nous avons effectué, il apparaît en définitive que seul le modèle de C. Almon constitue un système que satisfasse approximativement aux attendus théoriques et présente un réel intérêt pour l'étude économétrique de fonctions de demande détaillées." (p. 119). (From the study which we have done, it appears that definitely only the model of C. Almon offers a system which satisfies approximately theoretical expectations and is of real interest for the econometric study of detailed demand functions.) Elegant theoretical derivations, apparently, are of little help in finding adequate forms. Despite this relative success, there is a problem with the Almon suggestion, as we will see in section 20.3, where we will also see a way to fix it.

20.2. Slutsky Symmetry and Market Demand Functions

Just about the only useful non-obvious implication of the theory of the single consumer who maximizes utility subject to a budget constraint is the Slutsky symmetry shown in equation (20.2.1).

$$\frac{\partial x_i^k}{\partial p_j}+\frac{\partial x_i^k}{\partial y^k}x_j^k=\frac{\partial x_j^k}{\partial p_i}+\frac{\partial x_j^k}{\partial y^k}x_i^k \tag{20.2.1}$$

Here x_i^k is the consumption of product *i* by individual *k*, y^k is the nominal income of individual *k*, and p_j is the price of product *j*. A comparable relation, however, need not hold for the market demand functions, the sum over all *k* of individuals' demand functions. Summing the above equation over the individuals gives equation (20.2.2).

$$\frac{\partial \sum_k x_i^k}{\partial p_j}+\frac{\sum_k \partial x_i^k}{\partial y^k}x_j^k=\frac{\partial \sum_k x_j^k}{p_i}+\frac{\sum_k \partial x_j^k}{\partial y^k}x_i^k \tag{20.2.2}$$

which is in general not the same as – and does not simply – equation 20.2.3.

$$\frac{\partial \sum_k x_i^k}{\partial p_j}+\frac{\partial \sum_k x_i^k}{\partial \sum_k y^k}\sum_k x_j^k=\frac{\partial \sum_k x_j^k}{\partial p_i}+\frac{\partial \sum_k x_j^k}{\partial \sum_k y^k}\sum_k x_i^k \tag{20.2.3}$$

which is what Slutsky symmetry of the market demand functions would imply. Thus, strict micro theory does not imply Slutsky symmetry of market demand functions. Consequently, there is in general no "representative consumer." To suppose that market demand functions derived by maximizing the utility of this non-existent entity have "micro foundations" not enjoyed by functions not so derived is hardly respectful of micro theory. Rather, any market demand functions so derived

are on exactly the same theoretical footing as market demand functions made up without any reference to utility maximization. Both kinds of functions must meet the same "adequacy" criteria. With that point clearly established, we may, however, ask: Are there restrictive conditions under which equation (20.2.2) would imply equation (20.2.3)? One condition is, of course, that all individuals should have not only the same utility function but also the same income, and that the increase in aggregate income is accomplished by giving each the same increase. That condition is hardly interesting for empirical studies. A less restrictive condition is that the marginal propensity to consume a given product with respect to income should be the same for all individuals, or in effect, that the Engel curves for all products should be straight lines. If, for example,

$$\frac{\partial x_i^k}{\partial y^k} = a_i \tag{20.2.4}$$

Then the second term on each side of equation 20.2.1 can be factored to yield

$$\frac{\partial \sum_k x_i^k}{\partial p_j} + a_i \sum_k x_j^k = \frac{\partial \sum_k x_j^k}{\partial p_i} + a_j \sum_k x_i^k \tag{20.2.5}$$

This is exactly what equation (20.2.3) states, for in this case it makes no difference to whom the "infinitesimal" increase in income is given and

$$\frac{\partial \sum_k x_i^k}{\partial \sum_k y^k} = a_i \tag{20.2.6}$$

Now the assumption that all Engel curves are straight lines is generally contradicted by cross-section budget studies, even when one uses total expenditure in place of income in the Engel curves. (See, for example, Chao [1991] where Figure 2.2 shows Engel curves for 62 products). On the other hand, many products have virtually straight Engel curves over a considerable middle range of total expenditure where most households find themselves. Thus, one gets the impression that while Slutsky symmetry is certainly not a necessary property of market demand curves, *it probably does no great violence to reality to impose symmetry to reduce the number of parameters to be estimated*.

20.3. A Perhaps Adequate Form

The 1979 Almon article introduced a form with a multiplicative relation between the income terms and the price terms. Its general form is:

$$x_i(t) = (a_i(t) + b_i(y/P)) \prod_{k=1}^{n} p_k^{c_{ik}} \tag{20.3.1}$$

where the left side is the consumption per capita of product *i* in period *t* and $a_i(t)$ is a function of time. The b_i are positive constants. The *y* is nominal income per capita; p_k is the price index of product *k*; and *P* is an overall price index defined by

$$P = \prod_{k=1}^{n} p_k^{s_k} \tag{20.3.2}$$

where s_k is the budget share of product *k* in the period in which the price indexes are all 1, and the c_{ik} are constants satisfying the constraint

$$\sum_{k=1}^{n} c_{ik} = 0 \qquad (20.3.3)$$

Any function of this form is homogeneous of degree 0 in all prices and income and satisfies all of the properties set out in the first paragraph. It has three problems:

1. It is not certain that expenditures will add up to income.
2. There is no way to choose the parameters to guarantee Slutsky symmetry at all prices if we want to. We can, however, arrange to have symmetry in some particular base period. As long as the shares of various products in total expenditure do not change very much from those of that base period, we will continue to have approximate symmetry.
3. There are a lot of *c*'s to be estimated.

Problem 1 can be easily fixed by adding on a "spreader," that is, by summing all expenditures, comparing them with *y*, and allocating the difference in proportion to the marginal propensities to consume with respect to y at the current prices. The amount to be spread is usually small and the form with spreader has essentially the same properties as the form without, plus the adding up property. We need not complicate the mathematics here by adding the spreader, but in practice it should be added when the equations are used in forecasting.

Problem 2, in view of section 2, is more a cautionary note than a real problem. Symmetry in a base year is probably quite adequate.

Problem 3 – which occurs in all forms which provide for varying degrees of substitution and complementarity – can be quite severe. If we have 80 categories of expenditures, we have 6,400 *c*'s less the 80 determined by equation (10). If we have 20 years of annual data, we have 1,600 data points from which to determine these 5,600 parameters, or 3.5 parameters per data point! Clearly, we have to have employ some restrictions. Even if we had only one parameter per data point, we would probably want restrictions to insure reasonableness of the parameters. Indeed, the principal theoretical problem in consumption analysis is find ways to specify what is "reasonable."

Part of the solution of problem 3 can be found, if we wish, in the point noted in problem 2, namely that we can impose Slutsky symmetry at some prices. The Slutsky condition may be derived either from equation (20.2.1) or, more simply, by assuming that the compensating change in income is that which keeps *y*/*P* constant. Either approach gives as the symmetry condition equation (20.3.4):

$$\frac{c_{ij} x_i}{p_j} = \frac{c_{ji} x_j}{p_i} \qquad (20.3.4)$$

Multiplying both sides by $\frac{p_i p_j}{y}$ gives equation 20.3.5.

$$\frac{c_{ij}}{s_j} = \frac{c_{ji}}{s_i} \qquad (20.3.5)$$

If we then define

$$\lambda_{ij}=\frac{c_{ij}}{s_j} \tag{20.3.6}$$

then the form can be written as

$$x_i(t)=(a_i(t)+b_i(y/P))\prod_{k=1}^{n} p_k^{\lambda_{ik}s_k} \tag{20.3.7}$$

where

$$\lambda_{ij}=\lambda_{ji} \tag{20.3.8}$$

This restriction cuts the number of parameters by a half. That reduction is a big help but is clearly insufficient. Further help with this problem can be found through the idea of groups and subgroups of commodities. The accompanying box shows an example with fifteen basic commodity categories. These are subdivided into three groups and several categories which are not in any group. The first group is divided into two subgroups; the second, into one subgroup and a category not in the subgroup; the third group has no subgroup.

Illustration of Groups and Subgroups

	Product	*Group*	*Subgroup*
Food	1. Meat	I	A
	2. Fish	I	A
	3. Dairy products	I	A
	4. Cereal products	I	B
	5. Fruits and vegetables	I	B
	6. Other food products	I	B
Transportation	7. Automobiles	II	C
	8. Gasoline and oil	II	C
	9. Tires, batteries, repair	II	C
	10. Public transportation	II	
Clothing and Shoes	11. Clothing	III	
	12. Shoes	III	
No Group	13. Other durables		
	14. Other non-durables		
	15. Other services		

The idea of the Almon [1979] article was to assume that $\lambda_{ij}=\lambda_0$ if *i* and *j* are not members of the same group or subgroup, while if they are in the same group, *G*, $\lambda_{ij}=\lambda_o+\mu'_G$, and if they are in the same subgroup, *g*, of the group *G*, $\lambda_{ij}=\lambda_o+\mu'_G+\nu'_g$. Thus, there were as many parameters to estimate as there were groups + subgroups + 1. Estimation was fairly simple because, given a value of λ_0, estimation of the other parameters had to involve only products within the same group or subgroup. Several values of λ_0 were chosen, all equations estimated, and the value of λ_0 chosen which gave the best over-all fit.

The problem with this form was that products which had no natural partners with which to form a group all ended up either in very strange groups or, if they were given no group at all, all with nearly the same own price elasticity, namely $-\lambda_0$. It is often difficult to find groups for such goods as Telephone service, Medical service, Education, or Religious services. A specification which forces them all to have, for that reason, nearly the same own price elasticity is certainly inadequately flexible. An adequate form, it now seems, should allow every product to have its own own-price elasticity. We will then have as many price exponent parameters as there are products plus groups plus subgroups. A simple way to achieve this generalization is to introduce n parameters, λ_1, ..., λ_n, and use them to define the λ_{ij} as follows. If *i* and *j* are not members of the same group or subgroup, then

$$\lambda_{ij}=\lambda_i+\lambda_j \tag{20.3.9}$$

while if they are in the same group, *G*, $\lambda_{ij}=\lambda_i+\lambda_j+\mu'_G$, and if they are in the same subgroup, *g*, of the group *G*, $\lambda_{ij}=\lambda_i+\lambda_j+\mu'_G+\nu'_g$. The definitions apply only for *i* not equal to *j*. The λ_{ii} are each determined by equation (20.3.3), the homogeneity requirement.

Using these definitions, for product *i*, a member of group *G* and subgroup *g*, the equation becomes

$$x_i(t)=(a_i(t)+b_i(y/P))\prod_{k=1,k\neq i}^{n}p_k^{(\lambda_i+\lambda_k)s_k}\prod_{k\in G,k\neq i}^{n}p_k^{\mu_G's_k}\prod_{k\in g,k\neq i}^{n}p_k^{\nu_g's_k}p_i^{c_{ii}} \tag{20.3.10}$$

Equation (20.3.3) requires

$$\sum_{k\neq i}\lambda_k s_k+\lambda_i\sum_{k\neq i}s_k+\mu_G'\sum_{k\in G,k\neq i}s_k+\nu_g'\sum_{k\in g,k\neq i}s_k+c_{ii}=0 \tag{20.3.11}$$

If we solve this equation for c_{ii} and substitute in equation (20.3.10), we obtain, after a bit of simplification,

$$x_i(t)=(a_i(t)+b_i(y/P))(\frac{p_i}{P})^{-\lambda_i}\prod_{k=1}^{n}(\frac{p_k}{p_i})^{\lambda_k s_k}\left(\prod_{k\in G}(\frac{p_k}{p_i})^{s_k}\right)^{\mu'_G}\left(\prod_{k\in g}(\frac{p_k}{p_i})^{s_k}\right)^{\nu'_g} \tag{20.3.12}$$

where we have inserted the terms involving p_i/p_i into all of the products, because this term is always 1.0 no matter to what power it is raised. We can make the form even simpler by introducing price indexes for the group *G* and subgroup *g* defined by:

$$P_G=\left(\prod_{k\in G}p_k^{s_k}\right)^{1/\sum_{k\in G}s_k}\quad\text{and}\quad P_g=\left(\prod_{k\in g}p_k^{s_k}\right)^{1/\sum_{k\in g}s_k} \tag{20.3.13}$$

We then obtain equation (20.3.14)

$$x_i(t)=(a_i(t)+b_i(y/P))\cdot(\frac{p_i}{P})^{-\lambda_i}\prod_{k=1}^{n}(\frac{p_i}{p_k})^{-\lambda_k s_k}\cdot\left(\frac{p_i}{P_G}\right)^{-\mu_G}\left(\frac{p_i}{P_g}\right)^{-\nu_g} \tag{20.3.14}$$

where

$$\mu=\mu'\sum_{k\in G}s_k\quad\text{and}\quad\nu=\nu'\sum_{k\in g}s_k \tag{20.3.15}$$

This is the form for estimation. Note that it has one parameter, a λ, for each good, plus one parameter, a μ, for each group, plus one parameter, a ν, for each subgroup. Thus, it appears to have an adequate number of parameters. The Slutsky symmetry of (20.3.14) at the initial prices and income may be verified directly by taking partial derivatives of (20.3.14).

A special case of some interest arises when all the λ_i are the same and equal to $\lambda_0/2$, for in that case equation (20.3.14) simplifies to

$$x_i(t)=(a_i(t)+b_i(y/P))\left(\frac{p_i}{P}\right)^{-\lambda_i}\left(\frac{p_i}{P_G}\right)^{-\mu_G}\left(\frac{p_i}{P_g}\right)^{-\nu_g} \tag{20.3.16}$$

which is exactly the form suggested in the Almon [1979] article. Thus, the present suggestion is a simple generalization of the earlier one. In practice, there are apt to be a few commodities, such as Tobacco, Sugar, or Medical care which show so little price sensitivity that they cannot be fit well by this system. For them, we will assume that all the λ_{ij} in their rows and columns are 0. Note that this assumption is perfectly consistent with the symmetry of the lambda's. When there are such "insensitive" commodities in the system, equation (20.3.14) is modified in two ways. For these items, there are no price terms at all, while for other items the product term which in (20.3.14) is

shown with k running from 1 to n is modified so that k runs only over the "sensitive" and not the "insensitive" commodities.

It is useful in judging the reasonableness of regression results to be able to calculate the compensated own and the cross price elasticities. ("Compensated" here means that y has been increased so as to keep y/P constant.) Their derivation is straight-forward but complicated enough to make the results worth recording. In addition to the notation already introduced, we need

u_{ij} = the share in the base year of product j in the group which contains product i, or 0 if i is not in a group with j.

w_{ij} = the share in the base year of product j in the subgroup which contains product i or 0 if i is not in a subgroup with j.

μ_i = the μ for the group which contains product i, or 0 if i is not in a group. Note that μ_i is the same for all i in the same group.)

ν_i = the ν for the subgroup which contains product i, or 0 if i is not in a subgroup. (Similarly, note that ν_i is the same for all i in the same subgroup.)

L = The share-weighted average of the λ_i :

$$L=\sum_{k=1}^{n}\lambda_k s_k \tag{20.3.17}$$

The compensated own-price elasticity of product i is then:

$$\eta_{ii}=-\lambda_i(1-s_i)-L+\lambda_i s_i-\mu_i(1-u_{ii})-\nu_i(1-w_{ii}) \tag{20.3.18}$$

While the cross-price elasticity, the elasticity of the demand for good i with respect to the price of good j, is

$$\eta_{ij}=\lambda_i s_j+\lambda_j s_j-u_{ij}\mu_i+w_{ij}\nu_i \tag{20.3.19}$$

Two tables are produced by the estimation program. One shows, for each product, its share in total expenditure in the base year, the group and subgroup of which it is a member and its share in them, its λ and the μ and ν of its subgroups, its own price elasticity, and various information on the income parameters. Thus, it contains all the data necessary for calculating any of the cross elasticities. It is small enough to be reasonably reproduced. The other table shows the complete matrix of own- and cross-price elasticities. It is generally too large to be printed except in extract.

It should be noted that the complexity in estimating equation (20.3.14) comes from the term indicated by the product sign. Without this term, the equation could be estimated separately for each product or group of products. On the other hand, it is this term which gives Slutsky symmetry at the base point. If one did not care about this symmetry, then this term could omitted from the equation, with a great reduction in complexity in estimation. Once the programming has been done to estimate with this term, however, it is little trouble to use the program.

So far, we have said little about the "income" term, the term within the first parenthesis of equation (20.3.14). In the equations reported below we have used just a constant, real income per capita, the first difference of real income per capita, and a linear time trend. Furthermore, we have used the same population measure, total population, for computing consumption per capita for all items. The estimation program, however, allows much greater diversity. By use of adult-equivalency weights,

different weighted populations can be used for computing the per capita consumption of different items. Further, if the size distribution of income is known, it can be used to compute income-based indicators of consumption more appropriate to each item than just average income. Thus, the program allows a different income variable to be used for each consumer category. Finally, instead of just a linear time trend, one can use a "trend" variable appropriate to a particular category. For example, the percentage of the population which smokes could be used in explaining spending on tobacco. The estimation program allows for all these possibilities. On the other hand, in view of this diversity, it seemed pointless to try to place constraints on the parameters of the income terms to make the income terms add up to total income. Instead, in applying the estimated functions, one should calculate the difference between the assumed total expenditure and that implied by the equations and allocate it to the various items.

20.4. The Mathematics of Estimation

The function in equation (20.3.14) is nonlinear in all its parameters. In a system with 80 consumption categories there will be over 400 parameters involved in the simultaneous non-linear estimation. This size makes it worthwhile to note in this section some simplifying structure in the problem. All non-linear estimation procedures take some guess of the parameters, evaluate the functions with these values to obtain vectors of predicted values, $\hat{x}_i$, and subtract these from the vectors of observed values, x_i, to obtain vectors of residuals, r_i, thus:

$$r_i = x_i - \hat{x}_i \tag{20.4.1}$$

They then, in some way, pick changes in the parameters, and re-evaluate the function with the new values. The only difference in the various methods is how the changes in the parameters are picked. The Marquardt algorithm, which we use, is very nearly the same as regressing the residuals on the partial derivatives of the predicted values with respect to the parameters. It requires, in particular, these derivatives. For equation (20.3.14), they are reasonably easy to calculate if one remembers (or works out) the formula:

$$\frac{d\,a^x}{dx} = a^x \ln a \tag{20.4.2}$$

Where *ln* denotes the natural logarithm. Then the derivative of the demand for the i^{th} good with respect to its own λ is

$$\frac{\partial \hat{x}_i}{\partial \lambda_i} = \hat{x}_i \left(\ln \left(\frac{\prod p_k^{s_k}}{p_i} \right) \right) = \hat{x}_i \left(\sum s_k \ln p_k - \ln p_i \right) \tag{20.4.3}$$

and for j not equal to i

$$\frac{\partial \hat{x}_i}{\partial \lambda_j} = \hat{x}_i \ln \left(\frac{p_j}{p_i} \right) s_j = \hat{x}_i \left(\ln p_j - \ln p_i \right) s_j \tag{20.4.4}$$

And if i is a member of the group G

$$\frac{\partial \hat{x}_i}{\partial \mu_G} = \hat{x}_i \ln\left(\frac{P_G}{p_i}\right) = \hat{x}_i(\ln P_G - \ln p_i) \tag{20.4.5}$$

and if *i* is a member of the subgroup *g*

$$\frac{\partial \hat{x}_i}{\partial \nu_g} = \hat{x}_i \ln\left(\frac{P_g}{p_i}\right) = \hat{x}_i(\ln P_g - \ln p_i) \tag{20.4.6}$$

To explain the estimation process, we shall denote the vector of parameters of the "income-and-time term," the term preceding the first dot in equation (20.3.14), for product *i* by $\mathbf{a}_i$ and the vector of parameters of the "price term", the rest of the formula, by **h**. Thus, **h** consists of all values of λ, μ, and ν. Note that **h** is the same for all products, though a particular μ or ν may not enter the equation for a given commodity. If we let $\mathbf{A}_i$ be the matrix of partial derivatives of the predicted values for product *i* with respect to the $\mathbf{a}_i$ and similarly let $\mathbf{B}_i$ be the matrix of partial derivatives of the predicted values of product *i* with respect to **h**, and finally let $\mathbf{r}_i$ be the residuals, all evaluated at the current value of the parameters, then the regression data matrix, (**X**,**y**) in the usual notation, for three commodities is:

$$(X\,,\,y) = \begin{bmatrix} A_1 & 0 & 0 & B_1 & r_1 \\ 0 & A_2 & 0 & B_2 & r_2 \\ 0 & 0 & A_3 & B_3 & r_3 \end{bmatrix} \tag{20.4.7}$$

If we now form the normal equations **X'Xb = X'y** in the usual notation, we find

$$\begin{bmatrix} A_1'A_1 & 0 & 0 & A_1'B_1 \\ 0 & A_2'A_2 & 0 & A_2'B_2 \\ 0 & 0 & A_3'A_3 & A_3'B_3 \\ B_1'A_1 & B_2'A_2 & B_3'A_3 & \sum_{i=1}^{3} B_i'B_i \end{bmatrix} \begin{bmatrix} da_1 \\ da_2 \\ da_3 \\ dh \end{bmatrix} = \begin{bmatrix} A_1'r_1 \\ A_2'r_2 \\ A_3'r_3 \\ \sum_{i=1}^{3} B_i'r_i \end{bmatrix} \tag{20.4.8}$$

After initial values of the parameters have been chosen and the functions evaluated with these values and the sum of squared residuals (SSR) calculated, the Marquardt procedure consists of picking a scalar, which we may call *M*, and following these steps:

1. Compute the matrices of equation (20.4.8), multiply the diagonal elements in the matrix on the left by 1 + *M* and solve for the changes in the $\mathbf{a}_i$ and **h** vectors. Make these changes and evaluate the functions at the new values.
2. If the SSR has decreased, divide *M* by 10 and repeat step 1.
3. If the SSR has increased, multiply *M* by 10, go back to the values of the parameters before the last change, evaluate the functions again at these values, and repeat step 1.

The process is stopped when very little reduction in the SSR is being achieved and the changes in the parameters are small. (As *M* rises, the method turns into the steepest descent method, which can usually find a small improvement if one exists, while as *M* diminishes, the method turns into

Newton's method, which gives rapid convergence when close enough to a solution that the quadratic approximation is good.)

To economize on space in the computer and to speed the calculations, we can take advantage of the structure of the matrix on the left side of equation (20.4.8). To do so, let **Zi** be the inverse of $\mathbf{A_i'A_i}$. Then by Gaussian reduction (20.4.8) can be transformed into

$$\begin{bmatrix} I & 0 & 0 & Z_1 A_1' B_1 \\ 0 & I & 0 & Z_2 A_2' B_2 \\ 0 & 0 & I & Z_3 A_3' B_3 \\ 0 & 0 & 0 & \sum_{i=1}^{3} B_i' B_i - B_i' A_i Z_i A_i' B_i \end{bmatrix} \begin{bmatrix} da_1 \\ da_2 \\ da_3 \\ dh \end{bmatrix} = \begin{bmatrix} Z_1 A_1' r_1 \\ Z_2 A_2' r_2 \\ Z_3 A_3' r_3 \\ \sum_{i=1}^{3} B_i' r_i - B_i' A_i Z_i A_i' r_i \end{bmatrix} \tag{20.4.9}$$

The columns of the matrix on the left which are just columns of the identity matrix do not need to be stored in the computer. Instead, the program computes the terms in the last column of this matrix and in the vector on the right, stores only them, and at the same time builds up the sums in the lower right corner of the matrix and in the bottom row of the vector on the right. Once the matrix and vector of equation (20.4.9) are ready, the program solves the equations in the last row for **dh** and then substitutes back into the other equations to solve them for the $\mathbf{da_i}$.

The estimation program initializes the income parameters by regressing the dependent variables on the just the constant, income, and trend terms. Then all lambda's are started at .25 and all mu and nu at 0. The program was written in Borland C++ with a double-precision version of the BUMP library of matrix and vector objects and operators. The time required to do the estimation seems to be roughly proportional to the fourth power of the number of sectors. The work of evaluating the **B** matrices and taking **B'B** grows roughly with the cube of the number of sectors, so the time required for a single iteration grows with the cube of the number of sectors. The number of iterations, however, seems to grow at least linearly with the number of sectors, so the total time required should grow with the fourth power of the number of sectors. Thus, a 90-sector study can be expected to take about 16 times as long to estimate as a 45-sector study. This is roughly what we have experienced, with the 93-sector USA system requiring about 100 minutes and the 42-sector Spanish study five or six minutes on a 133 MHz pentium. The USA study required about 120 iterations. The big drops in the objective function started to appear after about 80 iterations.

20.5. Comparative Estimation for France, Italy, Spain, and the USA

To test how adequate this system is for representing the consumer behavior in a variety of countries, it has been estimated for France, Italy, Spain, and the USA. At the same time, so that the results would tell us something about the similarities and differences among these countries, the categories have been a been made as similar as possible. The categories, the groups, and the sub-groups are shown in the box to the right.

Groups and Subgroups for International Comparison

I. Food group
- 1 Cereal and bakery products
- A. Protein source subgroup
  - 2 Meat
  - 3 Fish & seafood
- 4 Dairy products
- 5 Fats & oils
- 6 Fresh fruit
- 7 Fresh vegetables
- 8 Sugar & sweets
- 9 Processed fruit and vegetables
- 10 Other prepared food, Pet food
- 11 Nonalcoholic beverages
- 12 Alcoholic beverages

II. Clothing group
- 14 Clothing and its cleaning and repair
- 15 Footwear and repair

III. House furnishing and operation group
- 18 Furniture
- 19 Floor coverings and textile products
- 20 Kitchen & hh appliances
- 21 China & glaswr, tablwr & utensils
- 22 Other non-durables and services
- 23 Domestic services
- 32 TV, radio, audio, musical instruments, computers

IV. Medical group
- 24 Drug preparations and sundries
- 25 Ophthalmic & orthopedic eqpt
- 26 Physicians, dentists, other
- 27 Hospitals, nursing homes

V. Transportation group
- A. Private transportation
  - 28 Vehicles
  - 29 Operation of motor vehicles
- 30 Public transportation

Ungrouped products
- 13 Tobacco
- 16 Tenant-occupied nonfarm space
- 17 Electricity, oil, gas, coal, water
- 31 Communication
- 33 Books & maps, Magazines and newspapers
- 34 Education
- 35 Recreational services
- 36 Personal care
- 37 Hotels & motels, restaurants
- 38 Other goods
- 39 Financial services and insurance
- 40 Other services

Extra American sectors not in European accounts
- 41 Food furnished to employees and food consumed on farn
- 42 Owner-occupied housing
- 43 Foreign travel
- 44 Imputed financial services

In using the word "test," I should make clear that I do not mean anysort of test of statistical "significance," which I regard as essentially meaningless here. The test is rather to see whether the system is flexible enough to fit the historical data with plausible values of the parameters. Moreover, it is not a test to see whether the program can find those reasonable values from the data alone. Whether or not that is possible depends upon what range of experience history has given us. It is often necessary to tell the program what values are plausible by soft constraints. The details of how that has been done are described in Appendix A on using the program. The Italian and Spanish data were for forty categories of consumer expenditures, most of them being exactly comparable. The French data were more detailed but were clearly based on the same statistical concepts and could be aggregated to match the Spanish and Italian. The three European datasets showed that the statisticians who had prepared them had been talking to one another and had achieved some degree of comparability. No such fundamental comparability infected the U.S. data. It was, however, available in much more detail than was the European, and in most cases, it was possible to match the European concept – as I understand it from the words in the definition – fairly closely. There were a few exceptions among foods. The Europeans had the following sectors:

6 Fruits and vegetables, except potatoes
7 Potatoes
9 Coffee, tea, and cocoa

I could not match these in the U.S. data but made up three sectors which at least keep the numbering the same for the other sectors. These were:

6 Fresh fruit
7 Fresh vegetables
9 Processed fruits and vegetables

Other known noncomparabilities included the Italians having no sector for Education but only one for text books, while the Spanish did not attempt to divide "all-included" vacation packages between

Transportation and Hotels and restaurants though the others did. Finally, the U.S. has four categories which have no corresponding component in the European accounts. First, and largest, is the imputed space-rental value of owner-occupied housing, which is seemingly not in the System of National Accounts (SNA) used by the Europeans. Second is Services rendered without payment by financial intermediaries (e.g. free checking accounts). The existence of these services is recognized by the SNA, but the European statistical offices (incorrectly) consider that all of these services are rendered to businesses, and thus appear in the intermediate part of the input-output table and do not enter GDP. Foreign travel shows up elsewhere in the European accounts and was not among the data series I had. Finally, Food furnished to employees or eaten on farms seems not to be part of the European system or appears directly in the various food categories. These extra sectors account for about 15 percent of American consumption. Within the forty more or less comparable sectors, the share of the American sectors in total consumption will average about 15 percent below the European.

The regressions were run from 1971 to 1994 (1993 for France.) It quickly became apparent that nearly all of the histories could be fit well, but often one or more of the parameters would have nonsense values. The income elasticity might turn out negative while there was a strong positive time trend. The own price elasticities, which should be negative, frequently turned out positive, perhaps at the same time that the income elasticity was negative. In short, the data were insufficiently varied to identify the parameters. Fortunately, the program used for the estimation (our creation) allowed for imposing "soft" constraints, which are essentially extra, artificial observations designed to tell the computer, before the estimation, what would be sensible regression coefficients. By using soft constraints, it is often possible to find equations with sensible coefficients which fit almost as well as the unconstrained equation. Except in Spain, where there was a drop in income in the mid 1980's before entry into the Common Market, time and income were very collinear, and it was necessary to softly constrain the time variable to be close to zero, though not exactly zero. In Spain, there was also a very soft constraint suggesting that the time trend coefficient should be small, but it was softer than in the other countries and consequently stronger time trends appear in the Spanish equations than in the others. In cases of products which evidently had strong time trends in tastes, such as fats and oils or tobacco, the soft constraint on the time trend was removed. Of course, the fact that soft constraints were used which were not identical in the different countries may reduce the comparability of the results. But it also shows that the system can be adapted to the situation in different countries.

Before commenting on the individual products, let us look at the results for the group parameters, as shown below.

Table 20.5.1 Comparison of Group and Subgroup Parameters for Four Countries

	$\mu's$ *across countries*					$\nu's$	
	Food	Clothing	Housing	Medical	Transport	Protein	PvtTrans
USA	0.25	0.96	-0.23	-0.26	0.06	-0.05	-0.54
Italy	-0.02	1.83	0.70	0.33	0.02	0.09	0.48
Spain	0.12	-0.34	0.21	0.00	0.07	0.20	-0.28
France	0.61	0.15	0.77	-1.36	0.07	0.57	-0.51

The components of the Food group did indeed turn out to be substitutes in the USA, Spain, and, especially, France. The protein sources were especially strong substitutes with one another in France and less so in Spain and Italy. In the USA, their special interaction was in the direction of complementarity. Buying cars and operating them were decidedly complements in the USA, Spain,

and France, but were rather strongly substitutes in Italy. The Italians may not, however, be totally crazy; automobile repair and new cars may indeed be substitutes. Shoes and Clothing turn out to be strongly substitutes in the USA and Italy, weak substitutes in France, and complements in Spain. he household furnishing and operating sectors showed considerable interaction, but were complements in America and substitutes in Europe. The medical sectors were complements in America and France and weakly substitutes in Italy. There is little interaction between public and private transportation in any of the countries.

Examining the individual sectors shows many interesting differences as well as some basic similarities among the countries. For each product, we will show the results of estimation for all four countries. The order of lines in these mini tables is USA, Italy, Spain, France. The sector titles have been left in the original language both to indicate the country and to describe as exactly as possible the content. On each line in the minitables for each product, you will find:

nsec	The sector number
title	The title of the product group in the language of the country
G	The number of the group in which the product is included. A 0 indicates that it was not in a group.
S	The number of the product's subgroup. A 0 means that it was not in a subgroup.
I	Inclusion code: 1 if the product was included in the estimation of the system, otherwise 0.
lamb	The value of lambda, λ, for this product.
Share	The share of this product in total consumption in the base year, the year when all the prices were equal to 1. Unfortunately for purposes of comparison of these shares, the base years were different: 1992 for the USA, 1988 for Italy, 1986 for Spain, and 1980 for France. These differences should have little effect on comparability except on these shares.
IncEl	The income elasticity, the percentage by which purchases of this item increase when income increases one percent.
Dinc	The ratio of the coefficient on the change in income to the coefficient on income.
Time%	The change in demand for the product due to the passage of one year (without change in income or price) expressed as a percentage of the average purchase.
PrEl	The elasticity of demand for the product with respect to its own price.
Err%	Standard error of estimate expressed as a percentage of the average value.
Rho	Autocorrelation coefficient of the residuals.

The commentary on each group also reflects looking at the graph of the fit in each country for each product. These graphs are, unfortunately, too space-intensive to print.

1. Bread and bakery products

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
1	Cereal and bakery produ	1	0	1	0.18	0.013	0.18	-0.60	0.01	-0.55	4.33	0.80
1	Pane e cereali	1	0	1	0.06	0.024	0.13	-1.59	0.00	-0.12	1.56	0.74
1	Pan y cereales	1	0	1	-0.12	0.026	0.18	-0.17	-0.26	-0.02	2.53	0.61
1	Pain et cereales	1	0	1	0.05	0.024	0.45	-0.03	0.00	-0.69	1.44	0.49

The Food group holds some striking similarities among the countries as well as big differences. *Bread and bakery products* (1) have seen virtually no growth in per capita consumption over the years covered here. Note, however, that the share is nearly twice as high in Europe as in America. The income elasticities, however, do not come out at zero but have been offset in the US and France by significant price elasticities. Italy shows both smaller income elasticities and small price elasticity, while in Spain the income elasticity comes out the same as that in the US but is offset by a negative trend of half a percent per year. The higher income elasticity in France may reflect the attractiveness of real croissants, brioche, and the like.

2. Meat

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
2	Meat	1	1	1	-0.16	0.018	0.03	0.47	-0.20	-0.19	3.98	0.74
2	Carne	1	1	1	0.05	0.056	0.23	2.00	0.00	-0.15	2.78	0.81
2	Carne	1	1	1	0.01	0.066	0.49	-1.00	-0.13	-0.21	2.94	0.50
2	Viandes	1	1	1	-0.80	0.062	0.54	0.00	0.00	-0.04	1.89	0.81

Only Spain has seen any noticeable growth in *Meat* (2) demand since 1980. It showed an income elasticity of .5 as did France, but Spain has had greater income growth. Both the US and Italy have very low income elasticities, though Italy has a positive "taste" term, while the US and Spain both show negative "taste" trends.

3. Fish and seafood

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
3	Fish & seafood	1	1	1	1.78	0.002	1.17	-0.07	-0.20	-2.12	8.90	0.52
3	Pesce	1	1	1	0.01	0.013	0.89	0.20	0.00	-0.15	4.17	0.83
3	Pescado	1	1	1	-0.02	0.024	0.35	-0.13	-0.34	-0.27	4.45	0.80
3	Poissons	1	1	1	0.00	0.008	1.58	0.11	0.00	-1.22	5.19	0.65

In striking contrast to Bread and Meat, *Fish and seafood* (3) shows strong income elasticities, above 1.0 in the USA, Italy, and especially France. Fish is definitely the food of the affluent in these countries, while it definitely is not in Spain, where consumption has declined steadily as income rose. Note, however, that the share of fish in the budget of Pedro was twice that of Pietro, three times that of Pierre, and twelve times that of Peter.

4. Milk and dairy products

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
4	Dairy products	1	1	1	-0.01	0.008	0.11	-0.13	-2.35	-0.34	6.53	0.70
4	Latte, formaggi	1	1	1	0.07	0.029	0.48	0.64	0.00	-0.20	1.97	0.69
4	Leche, queso y huevos	1	1	1	-0.10	0.033	0.07	-0.33	0.86	-0.18	3.68	0.79
4	Lait fromages et oeufs	1	1	1	0.04	0.025	0.83	0.04	0.00	-1.11	3.86	0.82

When it comes to *Milk and dairy products* (4), the US is the outlier. The European countries, where consumption runs from 2.5 to 3.3 percent of the total budget, have been increasing consumption steadily, while the USA is cutting back sharply from its already low share of .8 percent. The equation for France attributes the growth to income, the Spanish and Italian equations, more to taste trends. One may say that the American concern about cholesterol has not penetrated the European mind, or one may say that the American cheese industry has never approached the European in placing temptation in front of the consumer.

5. Fats and oils

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
5	Fats & oils	1	0	1	-0.08	0.002	0.06	-0.14	-0.37	-0.32	6.84	0.66
5	Oli e grassi	1	0	1	-0.04	0.008	0.07	2.43	-0.04	-0.03	2.85	0.73
5	Aceites y grasas	1	0	1	-0.08	0.011	0.06	-1.05	-0.54	-0.06	2.77	0.72
5	Huiles et graisses	1	0	1	-0.17	0.009	0.10	-0.62	-1.36	-0.53	2.42	0.34

6. Fruit and vegetables

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
6	Fresh fruit	1	0	1	0.15	0.003	0.86	-0.21	-2.48	-0.55	6.35	0.67
6	Frutta	1	0	1	0.05	0.043	0.26	-0.73	0.00	-0.11	1.64	0.04
6	Frutas y verduras	1	0	1	0.01	0.033	0.44	0.23	-1.04	-0.14	3.38	0.62
6	Fruits et legumes sauf	1	0	1	-0.45	0.025	0.20	0.52	0.00	-0.22	2.70	0.74

7. Fresh vegetables

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
7	Fresh vegetables	1	0	1	0.12	0.004	0.93	-0.20	-1.82	-0.52	8.96	0.77
7	Patate	1	0	1	-0.05	0.002	0.18	0.62	-0.01	-0.01	3.45	0.23
7	Patatas y tubérculos	1	0	1	-0.05	0.005	0.03	-1.07	-2.35	-0.10	6.60	0.70
7	Pommes de terre et autr	1	0	1	-0.63	0.002	-0.04	-6.46	-1.48	-0.09	7.63	0.67

Fats and oils (5) have uniformly low income elasticities and negative taste trends. Fruit has been declining in the US, while *Fruit and vegetables* (6), including canned and frozen, have been rising in Italy and stable in Spain and France. Recall that the sectoral definitions are not comparable here. The rest of the story for the US is found in *Fresh vegetables* (7), also in gentle decline, and in Processed fruits and vegetables (9), which also fails to show any growth. The total share for the US is 1.3 percent of the budget,only a half or a third of that of the European countries. That low share does not necessarily mean that we consume less than they do of these products. There are at least two other factors: larger total consumption and lower prices on agricultural products. The graphs for Potatoes show that the French are rapidly losing their appetite for French fries, as are the Spanish, while the Italians are not.

8. Sugar

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
8	Sugar & sweets	1	0	1	0.02	0.006	-0.01	2.83	-0.52	-0.41	5.92	0.47
8	Zucchero	0	0	0	0.00	0.003	0.10	7.14	-0.01	0.00	3.32	0.48
8	Azúcar	0	0	0	0.00	0.002	0.09	-0.65	-0.50	0.00	4.32	0.87
8	Sucre	1	0	1	-0.30	0.002	0.09	0.97	-2.83	-0.42	5.11	0.34

9. Coffee, tea & chocolate

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
9	Processed fruit and veg	1	0	1	-0.09	0.006	0.02	-0.27	-0.07	-0.30	6.16	0.86
9	Caffe, te, cacao	1	0	1	-0.03	0.005	0.43	-0.31	0.00	-0.03	2.73	0.67
9	Café, té y cacao	1	0	1	-0.01	0.006	0.03	-1.12	-0.44	-0.14	3.65	0.77
9	Cafe, thé	1	0	1	-0.44	0.006	0.24	-0.43	0.00	-0.27	6.84	0.90

Sugar (8) proved to be a problem in both Italy and Spain and was removed from the system in these two countries. The problem arose from substantial fluctuations in the price which had little effect on consumption. In the US and France, the system had no problem handling the product, and virtually identical price elasticities, -.4, were found. In France, however, there has been a strong trend away from sugar not seen in the US.

10. Other prepared foods

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
10	Other prepared food, Pe	1	0	1	-0.05	0.017	1.63	-0.72	0.00	-0.31	4.52	0.79
10	Altri generi alimentari	1	0	1	-0.03	0.006	0.64	-0.85	-0.01	-0.03	5.02	0.79
10	Otros alimentos	1	0	1	-0.02	0.007	0.34	-0.21	0.88	-0.13	2.08	0.71
10	Autres produits aliment	1	0	1	0.06	0.014	1.83	0.08	0.00	-0.74	3.25	0.49

The *Other prepared foods* (10) category , which includes the sauces, mixes, and just-run-it-in-the-microwave products have shown strong growth. For the USA, Italy, and France the equations attribute this growth to income, because of the aversion to time trends expressed in the soft constraints. In Spain, however, there were greater fluctuations in income and it was easier for the regression to distinguish time from income. It found that the income elasticity was actually fairly low, .34, and used a strong time trend, .9 percent per year, to account for the growth.

11. Soft drinks

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
11	Nonalcoholic beverages	1	0	1	0.11	0.009	0.77	1.35	-0.01	-0.50	4.83	0.60
11	Bevande analcoliche	1	0	1	-0.01	0.004	1.58	-1.37	-0.04	-0.05	7.68	0.83
11	Bebidas no alcohólicas	1	0	1	-0.12	0.005	1.12	-0.60	-0.66	-0.03	3.50	0.57
11	Boissons non alcoolisee	1	0	1	0.12	0.004	1.82	0.18	0.01	-0.83	10.11	0.77

The *Soft drink* industry (11), stagnant in the U.S. despite an income elasticity of .8 because of sharp price increases, has boomed in Italy and France, with income elasticity estimates of 1.6 and 1.8, respectively. The Spaniards have not been so easily seduced; they show an income elasticity of 1.1 and a negative time trend of .7 percent per year.

12. Alcoholic beverages

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
12	Alcoholic beverages	1	0	1	0.38	0.018	0.03	-0.02	-0.05	-0.73	3.55	0.84
12	Bevande alcoliche	1	0	1	-0.04	0.012	0.01	23.43	-0.90	-0.02	2.80	0.52
12	Bebidas alcohólicas	1	0	1	-0.07	0.014	0.09	4.24	-1.17	-0.08	5.70	0.72
12	Boissons alcoolisees	1	0	1	-0.01	0.024	0.17	-0.07	0.00	-0.64	2.36	0.75

Alcoholic beverage (12) sales have been static in the USA country and France, but declining in Italy and Spain. All countries showed very low income elasticities.

13. Tobacco

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
13	Tobacco	0	0	1	0.33	0.012	0.11	-0.02	-1.20	-0.48	3.16	0.21
13	Tabacco	0	0	0	0.00	0.016	0.03	37.79	1.03	0.00	7.10	0.86
13	Tabacos	0	0	1	0.06	0.016	0.34	-0.65	0.16	-0.09	3.00	0.53
13	Tabac	0	0	1	0.06	0.011	0.93	0.02	0.00	-0.17	2.82	0.76

The sharp decline in the use of *Tobacco* (13) in the USA has no parallel in Europe. In France, it was even rising up until 1992, showing an income elasticity of .93.

14. Clothing

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
14	Clothing and its cleani	2	0	1	0.00	0.044	1.36	-0.72	0.00	-0.30	1.31	0.46
14	Vestiario incl.riparazi	2	0	1	0.09	0.083	0.88	1.14	0.00	-0.53	2.57	0.64
14	Vestido	2	0	1	0.07	0.064	0.78	0.00	-0.74	0.00	2.97	0.46
14	Habillement sf chaus.	2	0	1	0.22	0.059	0.15	-0.13	0.00	-0.33	2.16	0.66

One of the surprises for me was the sad story of France in the consumption of *Clothing* (14). I had thought of the French as fashion conscious. Not at all, according to these equations. Clothing accounts for a smaller share in France than in any of the other European countries. The French income elasticity is only .15, against .8 for Spain, 1.4 for the U.S., and .9 for Italy, which has also the highest share of the budget going to clothes. Clearly it is the Italians who are the sartorially conscious nation.

15. Footwear

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
15	Footwear and repair	2	0	1	0.04	0.008	1.23	-0.03	0.02	-1.00	4.48	0.62
15	Calzature incl riparazi	2	0	1	-0.13	0.022	1.20	1.71	0.00	-1.40	6.28	0.86
15	Calzado	2	0	1	0.14	0.024	1.23	-0.21	-2.96	0.09	2.94	0.35
15	Chaussures y.c.reparat.	2	0	1	0.05	0.014	0.17	-0.32	0.00	-0.28	2.40	0.70

The same story holds for *Footwear* (15). The U.S., Spain, and Italy all came out with an income elasticity of 1.2, while in France it was only .2.

16. Rent

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
16	Tenant occupied nonfrm	0	0	1	0.05	0.046	0.98	-0.39	0.00	-0.21	2.20	0.71
16	Affitti per abitazioni	0	0	1	0.00	0.107	1.00	-0.42	0.00	-0.08	1.84	0.77
16	Alquileres y agua	0	0	1	0.03	0.111	0.53	-0.26	0.48	-0.05	6.98	0.96
16	Logement et l'eau	0	0	1	0.02	0.123	1.69	0.05	0.00	-0.12	2.55	0.69

Rent (16) on living quarters has risen steadily in all four countries; the income elasticities are 1.0 in

the U.S. and Italy; 1.7 in France; but only .5 in Spain. Rental payments did not fall during the Spanish slump of the early 1980's, so the equation attributes most of the growth to the time trend rather than to income.

17. Energy

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
17	Electricity, oil, gas,	0	0	1	-0.11	0.027	0.15	-0.08	0.00	-0.05	2.90	0.56
17	Combust.&energia elettr	0	0	1	-0.01	0.033	0.87	-0.07	0.00	-0.06	3.74	0.70
17	Calefacción y alumbrado	0	0	1	0.01	0.025	0.84	-0.28	2.08	-0.04	1.99	0.56
17	Electricité et combusti	0	0	1	0.00	0.052	0.53	-0.10	0.00	-0.11	3.95	0.45

Energy consumption (17) has been virtually constant in the U.S.; the equation found low income and price elasticities. All three European countries, but especially Spain, have seen significant growth. It is interesting that in Spain, where the equation was given less indication to avoid trend terms than in Italy and France, it used that extra freedom to get virtually the same income elasticity as was found in Italy, attributing the extra growth in Spain to the time trend.

18. Furniture

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
18	Furniture	3	0	1	-0.03	0.009	1.06	-0.03	-0.01	0.06	4.36	0.49
18	Mobili	3	0	1	0.04	0.028	1.57	0.26	0.00	-0.67	2.79	0.49
18	Muebles	3	0	1	0.08	0.021	1.38	-0.37	-1.95	-0.27	2.85	0.38
18	Meubles, tapis, y.c. re	3	0	1	0.57	0.031	0.43	-0.44	0.00	-1.23	4.67	0.66

The French are again the outlier in demand for *Furniture* (18). Spanish and Italian income elasticities are high and similar, 1.6 and 1.4 respectively; the US is a respectable 1.1; but France is only .4. Clearly the French have other priorities.

19. Carpets, curtains and household linens

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
19	Floor coverings and tex	3	0	1	0.06	0.004	1.64	-0.02	-0.17	-0.01	8.43	0.73
19	Biancheria e altri arti	3	0	1	-0.04	0.011	1.42	-0.30	0.00	-0.68	6.98	0.87
19	Artículos textiles	3	0	1	0.00	0.009	1.19	0.38	-1.08	-0.22	5.69	0.80
19	Art. de ménage en texti	3	0	1	0.00	0.007	0.00	-59.48	-0.03	-0.84	3.59	0.54

The story is the same for *Carpets, curtains, and household linens* (19). The French Income elasticity is exactly 0, while it is 1.2 to 1.6 for the other three countries.

20. Kitchen and household appliances

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
20	Kitchen & hh appliances	3	0	1	0.10	0.005	0.53	-0.06	-0.04	-0.05	5.31	0.65
20	Elettrodomestici	3	0	1	-0.37	0.012	1.14	0.64	0.00	-0.35	2.55	0.44
20	Electrodomésticos	3	0	1	-0.10	0.010	1.64	0.71	-1.27	-0.12	5.30	0.69
20	Ap. de cuis., de chauf.	3	0	1	-0.02	0.016	0.44	-0.31	0.00	-0.76	4.10	0.59

Kitchen and household appliances (20) show a similar pattern, except that both the U.S. and France have income elasticities close to .5, while in Italy and Spain, they are 1.1 and 1.6 respectively.

21. China, glassware and tableware

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
21	China & glaswr, tablwr	3	0	1	0.32	0.005	0.74	-0.03	0.08	-0.27	3.47	0.64
21	Cristallerie,vasellame	3	0	1	-0.66	0.006	1.02	0.21	-0.03	-0.10	3.28	0.61
21	Utensilios domésticos	3	0	1	0.04	0.005	0.17	-0.38	-1.65	-0.27	9.62	0.82
21	Verrerie, vaisselle et	3	0	1	-0.01	0.015	0.41	-0.17	0.00	-0.78	2.41	0.47

It is Italians and Americans who care about *China, glassware, and tableware* (21). The French have been particularly sensitive to the rising relative price of these products.

22. Other non-durables

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
22	Other non-durables and	3	0	1	0.59	0.019	0.52	-0.01	0.00	-0.57	2.05	0.45
22	Art.non dur. e servizi	3	0	1	-0.24	0.011	1.32	-1.92	0.01	-0.49	7.99	0.72
22	Mantenimiento	3	0	1	-0.10	0.015	0.95	0.12	-1.07	-0.11	3.82	0.70
22	Art. de ménage non-dur	3	0	1	0.01	0.018	1.15	0.02	0.00	-0.78	2.08	0.72

23. Domestic service

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
23	Domestic services	3	0	1	1.14	0.003	0.84	-0.02	-5.13	-1.08	10.45	0.80
23	Servizi domestici	3	0	1	0.03	0.025	1.44	0.33	0.00	-0.68	6.83	0.89
23	Servicio doméstico	3	0	1	-0.06	0.007	1.10	-1.06	-2.24	-0.17	7.91	0.79
23	Services domestiques	3	0	1	0.01	0.010	0.56	-0.04	0.00	-0.82	5.56	0.75

Domestic service (23) fell steadily in the U.S. from 1950 up to 1981, whereupon it suddenly stabilized and began a slow rise. Strikingly similar patterns appear in Spain and France, with low points in 1986 or 1987. Italy, by contrast, has shown strong growth all along, with an income elasticity of 1.4. All equations except the Spanish showed strong price elasticities.

24. Medicines

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
24	Drug preparations and s	4	0	1	0.08	0.018	1.42	-0.02	0.04	0.00	3.48	0.62
24	Medicinali e prod. farm	0	0	0	0.00	0.022	2.58	-1.01	-0.04	0.00	6.66	0.82
24	Medicamentos	4	0	0	0.00	0.016	3.03	-0.96	-1.42	0.00	15.31	0.84
24	Medicaments et autres p	0	0	1	1.06	0.021	1.50	0.21	0.00	-1.13	7.62	0.70

Medicines (24) have shown explosive growth in Europe. Note that all three European graphs ran off the standard scale. In Italy and France, this sector had to be thrown out of the system. The prices were rising and demand was soaring. Clearly, the problem was that the medicines were being paid for by third parties. The budget constraint had little relevance for the European buying medicine.

25. Ophthalmic and orthopedic devices

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
25	Opthalmic & orthopedic	4	0	1	0.07	0.003	2.35	-0.02	-0.34	0.02	9.62	0.73
25	Apparecchi e mater. ter	0	0	0	0.00	0.003	1.67	-0.70	-0.06	0.00	4.85	0.82
25	Aparatos terapéuticos	4	0	0	0.00	0.003	1.31	-1.28	0.20	0.00	9.59	0.82
25	Ap. et mat. therapeutiq	0	0	0	0.10	0.002	2.35	0.09	1.94	0.00	8.81	0.75

Similarly, *Ophthalmic and orthopedic devices* (25) could not be accommodated in the system in Italy and France. No price sensitivity but enormous income sensitivity is found in all countries.

26. Services of physicians, dentists and other medical professionals

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
26	Physicians, dentists, o	4	0	1	0.00	0.066	1.60	-0.02	0.00	-0.01	4.16	0.76
26	Serv. medici, infermier	4	0	1	0.26	0.024	1.29	-0.09	0.00	-0.44	4.85	0.79
26	Servicios médicos	4	0	0	0.00	0.010	1.80	-0.91	-0.70	0.00	8.58	0.81
26	Serv. des medecins infi	4	0	1	1.73	0.033	1.00	0.07	2.82	-1.25	5.02	0.66

Services of physicians, dentists, and other medical professionals (26) could be handled by the system in all countries. Only France, however, showed strong price sensitivity -- the U.S. showed none.

27. Hospitals

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
27	Hospitals, nursing home	4	0	1	0.07	0.063	1.40	-0.57	0.00	-0.07	3.54	0.86
27	Cure in ospedali&clinic	4	0	1	0.09	0.013	0.58	-2.59	0.00	-0.38	3.25	0.41
27	Atención hospitalaria y	4	0	0	0.00	0.006	0.89	-1.70	-0.83	0.00	5.07	0.63
27	Soins des hopitaux et a	4	0	1	0.91	0.018	0.32	-0.18	0.00	-0.11	2.93	0.53

The demand for Hospitals (27) has been decidedly more moderate than for the other members of the health care group. Only the U.S. shows an income elasticity greater than 1.0, and the French is down to .3.

28. Automobiles

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
28	Vehicles	5	2	1	0.16	0.043	1.32	-0.35	-0.01	-0.03	8.65	0.64
28	Acquisto di mezzi trasp	5	2	1	0.90	0.044	1.27	2.55	0.00	-1.17	5.98	0.46
28	Compra de vehículos	5	2	1	0.16	0.036	1.85	1.04	0.99	-0.04	8.06	0.42
28	Automobiles, caravanes,	5	2	1	0.17	0.041	1.36	-0.04	0.00	0.03	8.87	0.56

The income elasticity for *Automobiles* (28) is strong in all countries: 1.3 in the U.S., Italy, and France, and 1.8 in Spain -- and the Spanish equation has a 1 percent per year time trend on top of that income elasticity. The Spanish are plainly making up for lost time in equipping themselves to congest their streets and highways. Price sensitivity was slight, except in Italy.

29. Motor vehicle operation

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
29	Operation of motor vehi	5	2	1	0.27	0.060	0.67	-0.03	0.00	-0.20	2.20	0.42
29	Spese es. dei mezzi tra	5	2	1	-0.04	0.052	0.87	-1.26	0.00	-0.28	2.67	0.52
29	Gasto de uso de vehícul	5	2	1	0.10	0.075	1.10	-0.07	0.48	-0.06	2.15	0.54
29	Utilisation des véhicul	5	2	1	0.19	0.089	0.76	-0.13	0.00	-0.14	2.07	0.44

Motor vehicle operation (29), however, has an income elasticity of only .7 to .9 in three countries and 1.1 in Spain, where there is also a noticeable positive time trend.

30. Public transportation

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
30	Public transportation	5	0	1	0.17	0.008	0.43	-0.04	-0.05	-0.38	5.04	0.74
30	Acquisto serv. di trasp	5	0	1	0.00	0.016	0.90	-0.89	0.00	-0.09	2.43	0.39
30	Servicios de transporte	5	0	1	0.04	0.018	0.45	0.42	0.97	-0.13	1.95	0.16
30	Services de transport	5	0	1	0.07	0.022	0.89	-0.06	0.00	-0.24	2.76	0.74

Public transportation (30) claims a share of the consumer budget in Europe that is twice as large as the American share. Moreover, the income elasticities in Italy and France are twice what they are in the U.S. The U.S., however, is the most price sensitive, though the elasticity is only -.4.

31. Communications

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
31	Communication	0	0	1	0.20	0.019	1.52	-0.92	0.00	-0.35	2.71	0.80
31	Comunicazioni	0	0	1	1.16	0.011	1.55	-1.10	0.08	-1.22	5.84	0.58
31	Comunicaciones	0	0	1	-0.01	0.008	1.28	-0.33	2.76	-0.02	3.85	0.41
31	Telecommunications et p	0	0	1	0.04	0.015	3.75	0.08	0.01	-0.15	6.73	0.63

Communications (31) ran off the standard scale in all the European graphs. In Spain, where the equation was freer to use a time trend, the income elasticity came in at 1.3 with a strong positive trend of 2.8 percent per year added on to the income effect. In France, the equation attributed all the growth to income with a elasticity of 3.7! Since in both countries, much of the growth is attributable to the modernization of a once stodgy telephone monopoly, I suspect the Spanish equation is more appropriate.

32. TV, radio, audio, musical instruments and computers

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
32	TV, radio, audio, music	3	0	1	1.13	0.014	1.52	-0.01	0.00	-1.08	4.07	0.61
32	Apparecchi radio, tv, e	3	0	1	-0.46	0.040	1.87	-0.04	0.00	-0.14	2.17	0.44
32	Artículos de esparcimie	3	0	1	0.02	0.024	1.57	-0.04	0.00	-0.21	4.52	0.66
32	Radios, televiseurs, ar	3	0	1	0.02	0.036	1.52	-0.02	0.00	-0.69	4.13	0.57

The one and only runaway sector in the U.S. is *TV, radio, audio, musical instruments, and computers* (32). The enormous growth is, of course, in the computer component. We have not used the official "computer deflator" which would have made it grow even faster, but have left computers undeflated.

It is not clear to me whether or not computers are in this category in Europe. They probably are, because the category's share in total spending is considerably smaller here than in Europe. Even so, the income elasticity in the U.S. 1.5, the same as in Spain and France, and below Italy's 1.9. The key to the super fast growth is the relatively strong price elasticity, -1.1, coupled with the rapid decline of the relative price of these products.

33. Books, magazines and newspapers

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
33	Books & maps, Magazines	0	0	1	0.01	0.009	0.56	-0.05	0.01	-0.17	4.24	0.59
33	Libri, giornali e perio	0	0	1	0.01	0.017	0.73	0.61	0.00	-0.09	3.51	0.64
33	Libros, periódicos y re	0	0	1	0.06	0.017	0.60	-0.15	0.39	-0.08	2.56	0.64
33	Livres quotidiens et pe	0	0	1	0.04	0.015	0.54	0.13	0.00	-0.15	2.92	0.70

Despite the onslaught of electronic information and entertainment, *Books, magazines, and newspapers* (33) have hung on to their absolute level of sales, though they have been losing share in the consumer's dollar, as appears from the modest income elasticities of .6 or .7, the highest being in Italy, which, with Spain, has the highest share of the consumer's budget, almost twice that in the USA.

34. Education

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
34	Education	0	0	1	0.04	0.022	0.92	-0.02	0.00	-0.20	2.86	0.84
34	Libri per l'istruzione	0	0	1	-0.03	0.008	1.49	0.05	0.01	-0.05	6.22	0.85
34	Enseñanza	0	0	1	0.01	0.008	0.67	-1.68	-0.72	-0.04	4.82	0.66
34	Enseignement	0	0	1	-0.01	0.004	0.37	1.17	2.16	-0.10	16.51	0.77

Lest that dismal comparison leaves you a bit embarrassed to be American, take heart from *Education* (34), for which the American budget share is nearly three times that of the European. Alas, however, the difference is much affected by accounting conventions. In the U.S., all of the endowment income of schools and colleges counts as consumption expenditures on education. The income elasticity in the USA is .9, .7 in Spain, and a paltry .4 in France, where the budget share is less than a fifth of that here. The French expect the state to cover all the costs of education.

35. Recreational services

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
35	Recreational services	0	0	1	0.07	0.031	1.84	-0.64	0.00	-0.22	2.49	0.71
35	Spettacoli, serv. ricre	0	0	1	-0.02	0.023	1.05	-0.88	0.00	-0.06	3.13	0.73
35	Servicios de esparcimie	0	0	1	-0.03	0.019	0.67	-0.43	-1.01	0.00	2.36	0.66
35	Serv. de loisir, specta	0	0	1	0.08	0.019	1.08	0.03	0.00	-0.18	2.46	0.77

Recreational services (35), which includes spectator sports, have been a growth industry everywhere except in Spain. The U.S. leads with an income elasticity of 1.8, and a budget share of 3.1 percent. Italy and France also have elasticities above 1 and budget shares of 2.3 and 1.9 percent respectively. In Spain, however, the income elasticity was only .7 and there was a negative time trend of a percent

per year. My suspicion is that the great national spectator sport of bull fighting is to some extent losing its hold on the imagination of young, urban Spaniards.

36. Personal care articles and services

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
36	Personal care	0	0	1	0.28	0.015	0.72	-0.04	0.00	-0.43	3.93	0.82
36	Beni e servizi igiene p	0	0	1	0.04	0.030	1.21	-0.59	0.00	-0.11	6.07	0.91
36	Cuidados y efectos pers	0	0	1	0.00	0.014	0.76	-0.98	0.88	-0.03	2.80	0.37
36	Soins personnels, art.	0	0	1	-0.01	0.015	1.38	0.05	0.00	-0.10	4.30	0.83

Personal care articles and services (36), which includes everything from tooth paste to hair salons, has been a growth industry in Europe, with income elasticities of 1.4 in France and 1.2 in Italy, in contrast to .7 in America. The Spanish equation used its greater freedom to use a trend term to find almost exactly the American income elasticity but add to it a time trend of .9 percent per year.

37. Hotels and restaurants

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
37	Hotels & motels, restau	0	0	1	0.07	0.053	0.90	-0.03	0.00	-0.22	2.53	0.63
37	Spese alberghi e pubbl.	0	0	1	0.09	0.095	1.03	-0.25	0.00	-0.15	1.47	0.37
37	Restaurantes cafés y ho	0	0	1	0.08	0.013	0.88	-1.14	0.59	-0.11	3.32	0.43
37	Hotels, cafés, restaur.	0	0	1	0.10	0.065	0.82	0.00	0.00	-0.20	1.95	0.51

The *Hotels and restaurant* (37) business enjoys fairly good income elasticities (.8 to 1.0) in all four countries.

38. Other goods

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
38	Other goods	0	0	1	0.08	0.031	1.54	-0.02	0.00	-0.23	3.43	0.54
38	Altri beni	0	0	1	0.72	0.031	1.83	-0.25	0.00	-0.75	3.94	0.62
38	Otros artículos n.c.o.p	0	0	1	0.04	0.153	1.34	-0.30	0.19	-0.05	8.11	0.92
38	Autres articles	0	0	1	0.34	0.016	0.11	-2.44	0.00	-0.43	9.51	0.63

The *Other goods* category (38) seems to be totally non-comparable across countries. Just the fact that it accounts for 15 percent of the Spanish budget and 1.6 percent of the French budget indicates that the contents of the sector must be quite different.

39. Financial services and insurance

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
39	Financial services and	0	0	1	0.05	0.039	1.60	-0.53	0.00	-0.20	2.57	0.67
39	Serv. finanz. e assicur	0	0	1	1.04	0.005	1.76	-0.69	0.18	-1.10	11.54	0.87
39	Servicios financieros n	0	0	1	0.12	0.002	1.72	-0.82	-4.83	-0.15	19.92	0.85
39	Services financiers n.d	0	0	1	0.10	0.007	3.67	0.06	0.09	-0.21	9.28	0.54

Financial services and insurance (39) has been a major growth industry in France and Italy, where it found income elasticities of 3.7 and 1.8, respectively. What looks like a change of definition has

dominated the Spanish series. In America, this is a much more mature industry with roughly ten times the budget share it carries in Europe. (The services rendered without payment by financial intermediaries are not included here.)

40. Other services

nsec	title	G	S	I	lamb	share	IncEl	DInc	time%	PrEl	Err%	rho
40	Other services	0	0	1	0.07	0.064	1.27	-0.03	0.00	-0.22	2.87	0.61
40	Altri servizi	0	0	1	-0.07	0.008	1.16	-1.06	0.02	-0.01	7.90	0.88
40	Otros servicios n.c.o.p	0	0	1	0.10	0.033	0.55	-0.07	2.23	-0.12	3.61	0.76
40	Autres services n.d.a	0	0	1	0.07	0.022	0.88	-0.06	0.00	-0.18	2.86	0.75

Other services (40) are also much more important in the US than in Europe, but continue to have a higher elasticity here (1.3) than in France (.9) or Spain (.5). The Italian elasticity is high (1.2) but on a very small base, only 0.8 percent of the budget.

It would be safe and politic to conclude that this comparison has shown that the new functional form is capable of representing a variety of behavior, including significant substitution and complementarity. While that is, from a technical point of view, the most important conclusion, I cannot pass up the temptation to try to picture the national characters as they appear from these estimates. This venture is especially dangerous since citizens of all the four countries may be readers. Please take no personal offense.

The American has enough if not too much to eat, has become diet conscious and is cutting down on cholesterol but is, sad to say, bothering less and less to prepare fresh fruits and vegetables. He would like to eat more fish but is very sensitive to its price. He has no particular interest in more alcohol; soft drinks are sort of a necessity, not a special treat, and smoking is just a way to make yourself into a social outcast. Ms. America is very concerned about how she and her family are dressed. Housing and furnishing and equipment for the house are important, but using more energy for running the home is a matter of no interest. More domestic help for the working woman is becoming important again. The nation has gone bonkers over home computers. Every child of any age must have one. Books? Well, of course, a few books. But sports, concerts, plays, skiing, sailing, any kind of recreation, that's what America is all about. Relative to the Europeans, the American is not starved for medical care, and growth in this area has been less here than there. Automobiles are important but not much of a class symbol; operating them is just a necessity. Private education and tuition at public universities is a serious matter. Use of communication has grown because of the declines in its price. Public transport is to be avoided.

The Italian is outstanding among Europeans for dressing well. He is proud of his country's cucina, and would gladly eat more cheese, fish, and, above all, soft drinks; but he is losing interest in vino. Of pasta, he has, thank you, enough. Eating out at a good trattoria, ah, that's worth the price. He continues to puff away on his cigarette just to show his defiance of statistics. In an energy-poor country, he wants more electricity and fuel. Signora is concerned not only with dressing the family well but is especially concerned to have a well-furnished house, refined furniture, attractive carpets and linens, and a bit of style and elegance in china and crystal. She wants appliances to help her with the house work. But most of all she wants domestic help. Dispensing with domestic servants was a modern idea that never crossed her sensible mind. The family has been upgrading its car,

especially because prices have been coming down relative to other goods. The modest motor scooter is giving way to the even noisier motorcycle. But public transport is still a respectable way to get around. Especially if you have a portable telephone -- and who doesn't? Yes, computers and audio equipment have caught on fast, just not to the mania level of the Americans. Reading the newspaper is very important, and books still hold more allure than in any of the other three countries. One might suppose that just watching Italian politics would provide spettacoli enough, but no, recreation is high on the list of priorities. Socialized medicine led to explosive growth of spending on medicine but not on doctors, and certainly not on hospitals.

The Spanish are the newly rich of Europe. And the riches come after a period of declining income in the early 1980's. They have increased meat consumption in the last five years. Eating out is great, but please, no more fish! And less potatoes and wine. But an extra cigarette, por favor. Clothing is a good thing to economize on, as are shoes, though, of course, as income goes up you should look just a little better. Pretty much the same goes for furniture, rugs, and linens. China and glassware are an especially good place to economize when your income rises. One good place to put some of the savings on these goods is into more and better appliances along with the electricity to run them. Indeed, electricity is showing such a growth that one becomes suspicious that air conditioning might be catching on. But top priority for these savings is the car. No other of our countries is close to the Spanish income elasticity for cars. And if you have bought the car, then you have to drive it. But the new high-speed rail lines are making public transportation competitive again. Right behind the car in priority comes "recreational equipment" which seems to correspond to home electronics, including possibly the computer. Never mind, however, about those recreational services that everybody else is so crazy about. Just living in Spain is recreation enough. As in all three European countries, medical expenditures have skyrocketed: medicines, therapeutic devices, and services of doctors. Even hospital services have seen some rise.

Now the French seem utterly indifferent to improving, when income rises, how they are dressed or to how their house is furnished or to what sort of china or glassware they use, but not to what they eat and drink. Increase the French family's income by one percent, and it will spend .8 percent more on dining out, .5 percent more on meat, 1.6 percent more on fish and those delicious shellfish, .8 percent more on cheese, 1.8 percent more on candy and "other" prepared foods, 1.8 percent more on soft drinks, including, of course, mineral water, and even a bit more, .2 percent, on wine. If the French don't uphold their reputation as the fashion center of Europe, they are certainly les gourmands of the continent. It must be added that they are cutting back sharply on sugar and potatoes. Alone among the four countries, they are increasing their use of tobacco. They attach less priority to buying household appliances than to increasing meat consumption. A pleasant effect of that indifference is that energy consumption has remained stable. They have relatively little interest in new cars, relative, that is, to their neighbors in Spain and Italy, and expenditures on operating the cars are correspondingly stable. Public transport is more income elastic than is operation of cars. Besides their interest in food, they spend added income on personal care, on home electronics and recreational equipment, on cultural and sporting events, on financial services, and on communications. As in the other European countries, the large shares of increased income have gone to -- or come in the form of -- medicines, therapeutic devices, and services of doctors.

Well, it seems we are not all the same. There do appear to be national differences that go beyond language. For the present purposes, however, the important point is that this new functional form seems to be able to work well in what turns out to be a surprising variety of situations. Perhaps it is

adequate.

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Appendix A. Use of the Estimation Program

The Interdyme system has a module, PADS.CPP, for calculation of simulations with the PADS system. Several versions of PADS are offered, and the user should look at the comments in the program to see which is the appropriate version. The estimation program, known as *Symcon*, produces output compatible with input requirements of this module. *Symcon* has two control input files, GROUPS.TTL and SOFTCON.DAT, and several data matrices, CONSUM.DAT, PRICES.DAT, CSTAR.DAT, POPUL.DAT, and TIME.DAT.

The GROUPS.TTL file, as the name suggests, defines the groups. It also specifies which categories are sensitive and which insensitive to price, which weighted population, which income variable, and which trend variable is to be used by each category. This file for the Spanish study is shown in the box below. Its first column consist of simply the integers from 1 to n, the number of categories of consumption. The second column carries the number of the group in which the category falls, or a zero if it is not assigned to a group, and the third column carries the number of the subgroup to which the category belongs or a zero if it belongs to none. The fourth is the number of the weighted population to be used for the item, the fifth is the number of the "income" (or Cstar) series to be used, the sixth is the number of the "trend" series to be used, and the seventh is a 1 if the category is a regular, price-sensitive commodity or a 0 if it is not. Although conceptually we have thought of neatly defined groups and subgroups strictly within the groups, the computer program makes no effort to enforce this tidy structure. It is possible to form "subgroups" with categories drawn from more than one group.

The second major control file is SOFTCON.DAT, which gives soft constraints for the various equations. It is, in fact, hardly to be expected that all parameters would come out with reasonable values when so many of the variables have similar trends. Thus the use of soft constraints on the coefficients is an integral part of the estimation process. The estimation program allows the user to specify the desired value of any parameter except the constant term and to specify a "trade-off parameter" to express the user's trade-off between closeness of fit and conformity with desired values of the parameters. In these studies, I began with constraints saying that I wanted the time trends to be close to zero. I then worked on the income elasticities to get them all positive; for some products, that meant relaxing the soft constraint on the time trend. Then I added soft constraints to make the own price elasticities all negative. Finally, some of the coefficients on the change in income had to be constrained to keep them from being more negative that the income term is positive.

The SOFTCON.DAT file for Spain is shown in a box below. For each product, there can be specified desired values of the income elasticity, the change in income in elasticity units, the time trend as a percent of the base year (1988) value, *lambda*, and the *mu* and *nu* of the group and subgroup. The table shows for each of these a pair of numbers, the desired value and the trade-off parameter. If the trade-off parameter is 0, the desired value has no effect on the estimation. The higher the parameter, the stronger the constraint relative to the data. A value of 1.0 for the trade-off parameter gives about equal weight to the constraint and to the data. Constraints on *mu* and *nu* values can be specified on the line for any member of the group or subgroup, but I have always placed them on the line of the first item in the group or subgroup. This table is, in fact, precisely the way the constraints are entered into the program; the table shows the contents of the file SOFTCON.DAT, which is read by the estimation program.

The GROUPS.TTL File for Spain

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```
# Groups.ttl.  Columns are
# 1 The consumption category number
#    2 The group number
#       3 The subgroup number
#          4  Which weighted population number to be used with this category
#             5 Which Income (Cstar) variable
#               6 Which Trend variable
#                 7 Use price terms ( 1 = yes, 0 = no)
#                   8 The title of the category
 1  1  0  1  1  1 1 Pan y cereales
 2  1  1  1  1  1 1 Carne
 3  1  1  1  1  1 1 Pescado
 4  1  1  1  1  1 1 Leche, queso y huevos
 5  1  0  1  1  1 1 Aceites y grasas
 6  1  0  1  1  1 1 Frutas y verduras
 7  1  0  1  1  1 1 Patatas y tubérculos
 8  0  0  1  1  1 0 Azúcar
 9  1  0  1  1  1 1 Café té y cacao
10  1  0  1  1  1 1 Otros alimentos
11  1  0  1  1  1 1 Bebidas no alcohólicas
12  1  0  1  1  1 1 Bebidas alcohólicas
13  0  0  1  1  1 1 Tabacos
14  2  0  1  1  1 1 Vestido
15  2  0  1  1  1 1 Calzado
16  0  0  1  1  1 1 Alquileres y agua
17  0  0  1  1  1 1 Calefacción y alumbrado
18  3  0  1  1  1 1 Muebles
19  3  0  1  1  1 1 Artículos textiles
20  3  0  1  1  1 1 Electrodomésticos
21  3  0  1  1  1 1 Utensilios domésticos
22  3  0  1  1  1 1 Mantenimiento
23  3  0  1  1  1 1 Servicio doméstico
24  4  0  1  1  1 0 Medicamentos
25  4  0  1  1  1 0 Aparatos terapéuticos
26  4  0  1  1  1 0 Servicios médicos
27  4  0  1  1  1 0 Atención hospitalaria y seguro médico privado
28  5  2  1  1  1 1 Compra de vehículos
29  5  2  1  1  1 1 Gasto de uso de vehículos
30  5  0  1  1  1 1 Servicios de transporte
31  0  0  1  1  1 1 Comunicaciones
32  3  0  1  1  1 1 Artículos de esparcimiento
33  0  0  1  1  1 1 Libros, periódicos y revistas
34  0  0  1  1  1 1 Enseñanza
35  0  0  1  1  1 1 Servicios de esparcimiento
36  0  0  1  1  1 1 Cuidados y efectos personales
37  0  0  1  1  1 1 Restaurantes cafés y hoteles
38  0  0  1  1  1 1 Otros artículos n.c.o.p.
39  0  0  1  1  1 1 Servicios financieros n.c.o.p.
40  0  0  1  1  1 1 Otros servicios n.c.o.p
41  0  0  1  1  1 1 Viajes turísticos todo incluido
```

The SOFTCON.DAT File for Spain

```
sec  Title                      Income      DIncome     Time        lambda     mu        nu
1 Pan y cereales               0     0      0     1     0   .5      .2   5.   .1  1.
2 Carne                        0     0      0     0     0   .5      .2   5.    0  0    .2 1.
3 Pescado                      0     0      0     0     0   .5      .2   5.
4 Leche, queso y huevos        .1    1.     0     1.    0  1.0      .2   5.
5 Aceites y grasas             .1    1.   -.04    1.    0   .0      .2  10.
6 Frutas y verduras            0     0      0     0     0   .5      .2   5.
7 Patatas y otros tubérculos   .0    1.   -.02    1.    0   .5      .2  10.
8 Azúcar                       .1    1.   -.06    1.    0   .5
9 Café, té y cacao             .05   1.   -.03    1     0   .5      .2  10.
10 Otros alimentos             0     0      0     0     0   .5      .2  10.
11 Bebidas no alcohólicas      0     0      0     0     0   .5      .2   5.
12 Bebidas alcohólicas         .05   1.     0     0     0   .5      .2  10.
13 Tabacos                     0     0     -.2    1.    0   .5      .2  20.
14 Vestido                     0     0      0     0     0   .5      .2   5.
15 Calzado                     0     0      0     0     0   .5      .2   5.
16 Alquileres y gasto de agua  0     0     -.05   1.    0    1.
17 Calefacción y alumbrado     0     0      0     0     0   .5      .2   5.
18 Muebles                     0     0      0     0     0   1.      .2   5.   .1  1.
19 Art. textiles para el hogar 0     0      0     0     0   .5      .2  10.
20 Electrodomésticos           0     0      0     0     0   .5      .2  10.
21 Utensilios domésticos       .1    1.   -.06   1.    0   .5      .2   5.
22 Mantenimiento               0     0      0     0     0   .5      .2   5.
23 Servicio doméstico          0     0    -1.2    1     0   .1      .2   5.
24 Medicamentos                0     0      0     0     0   1.
25 Aparatos terapéuticos       0     0      0     0     0   1.
26 Servicios médicos           0     0      0     0     0   .5
27 Atención hospitalaria y smp 0     0      0     0     0   .5
28 Compra de vehículos         0     0      0     0     0   .5      .2  20.  .1   5.  -.3 1.
29 Gasto de uso de vehículos   0     0      0     0     0   .5      .2   5.
30 Servicios de transporte     0     0      0     0     0   .5      .2  10.
31 Comunicaciones              0     0      0     0     0   .5     .2  10.
32 Artículos de esparcimiento  0     0      0     0     0   .5      .2  10.
33 Libros, periódicos y revist 0     0    -.1     1.    0   .5      .2   5.
34 Enseñanza                   0     0      0     0     0   .5      .2  10.
35 Servicios de esparcimiento  0     0    -.30    1.    0   .5      .2   5.
36 Cuidados y efectos personal 0     0      0     0     0   .5      .2  10.
37 Restaurantes cafés y hotele 0     0      0     0     0   .5      .2  10.
38 Otros artículos n.c.o.p.    0     0    -.40    1.    0    2.     .2  10.
39 Serv. financieros n.c.o.p.  0     0     -1.4   1.    0   2.0     .2   5.
40 Otros servicios n.c.o.p     0     0      0     0     0   .5      .2   5.
41 Viajes turís. todo incluido 1.0   1.     0     0     0   .5      .2   5.
```

The CONSUM.DAT file begins with some dimensions and dates and then contains the data on consumption in almost exactly the form in which it would be written by the *G7* command “matty”. The layout is shown in the above box for the Spanish case; the “...” show where material has been cut out of the file to make it fit on the page. Notice the four numbers with which it begins. Each should be on its own line. Then come the data, with 20 series at a time across the "page". Comments may be introduced in the data by beginning the line with a #.

The CONSUM.DAT File for Spain

```
42 Sectors
24 years of data
1971 First year
1986 Base year
# Consumption, constant 1986 prices, total (not percapita)
# Date kcpi1 kcpi2 kcpi3 kcpi4 ... kcpi20
# 70.000 499.340 1021.118 549.566 467.982 ... 164.161
71.000 490.641 1022.712 571.055 466.638 ... 175.657
72.000 506.050 1017.367 574.642 472.211 ... 201.513
73.000 548.905 1182.404 582.436 498.514 ... 210.624
74.000 554.821 1338.080 536.756 562.671 ... 202.528
... ... ... ... ... ...
94.000 599.644 1666.945 606.117 735.611 ... 289.384
# Date kcpi21 kcpi22 kcpi23 kcpi24 ... kcpi40
# 70.000 109.692 286.789 184.407 272.749 ... 68.330
71.000 117.373 299.460 188.615 313.366 ... 74.745
72.000 134.650 342.775 189.809 337.034 ... 79.912
73.000 140.738 351.768 187.654 340.828 ... 85.539
74.000 135.328 350.958 180.607 370.052 ... 86.795
... ... ... ... ... ...
94.000 121.689 412.728 183.031 780.092 ... 165.796
# Date kcpi41 kcpi42
70.000 34.073 298.342
71.000 38.251 334.738
72.000 45.404 395.237
73.000 48.923 425.776
74.000 55.430 479.674
... ... ...
94.000 48.288 934.576
The ... indicate where data have been removed to fit the into this box.
```

Exactly the same format is followed for the PRICES.DAT file, which give the price indexes, except that the four numbers at the top are omitted. The CSTAR.DAT file, which gives the income series, begins with the number of such series. It then has these series arranged in columns. It has one extra year of data at the beginning so that the first difference of income can be calculated. The POPUL.DAT file is very similar; it begins with an integer giving the number of populations, followed by data in the same format. It also has the extra year at the beginning. Finally the TEMPI.DAT file gives various series which may be used as the time trend. Like the POPUL.DAT file, it has the number of series at the beginning but does not have the extra year of data at the beginning.

Once the files GROUPS.TTL, CONSUM.DAT, PRICES.DAT, CSTAR.DAT, TEMPI.DAT, and SOFTCON.DAT are ready, the program is run by the command "symcon [n]" from the DOS prompt. The optional parameter, n, is the number of iterations to be run before turning over control to the user. Thus *Symcon* will run only 1 iteration and then give the user the option of quitting (by tapping 'y') or continuing the Marquardt process another iteration. If the command given is "symcon 40",

then 40 iterations are automatically run without pausing for user input. To check that data have been read correctly, use "symcon d". (The d is for "debug".)

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